

## INTRODUCTION

- NEED A 'CONCEPTUAL FRAMEWORK' TO MAKE DECISIONS
- FRAMEWORK IS PROBABILITY
- 'STATISTICAL MODELS OF REALITY'
- DISCRETE (BINOMIAL, POISSON)  
CONTINUOUS (NORMAL)

## TERMINOLOGY

- *SAMPLE SPACE*: SET OF ALL POSSIBLE OUTCOMES
- *EVENT*: ANY SET OF OUTCOMES

## EXAMPLES

SAMPLE SPACE: ONE DICE {1, 2, 3, 4, 5, 6}

EVENT: 3

SAMPLE SPACE: BIRTHS {ALL BIRTHS}

EVENT: LOW BIRTH WEIGHT (<2500 GRAMS)

## TWO DEFINITIONS OF PROBABILITY

\* CLASSICAL (NO NEED FOR DATA)

$P(A)$  = NUMBER OF WAYS AN EVENT CAN OCCUR DIVIDED BY  
NUMBER OF ALL POSSIBLE EVENTS

SAMPLE SPACE:  $\{1, 2, 3, 4, 5, 6\}$

EVENT: 3

$$P(A) = 1 / 6 = 0.167$$

SAMPLE SPACE:  $\{H, T\}$

EVENT: H

$$P(A) = 1 / 2 = 0.5$$

## RELATIVE FREQUENCY (DATA-BASED)

$P(A)$  = COUNT OF THE NUMBER OF EVENTS THAT OCCUR  
DIVIDED BY THE COUNT OF ALL POSSIBLE EVENTS

SAMPLE SPACE: {ALL BIRTHS}

EVENT: LOW BIRTH WEIGHT INFANT

$$P(A) = 489 / 4119 = 0.119$$

(NYS DATA BASED, TWO HOSPITALS IN 2000)

SAMPLE SPACE: {ALL BIRTHS}

EVENT: MALE INFANT

$$P(A) = 1,927,054 / 3,760,358 = 0.51247$$

(ROSNER, TABLE 3.1)

## TWO TYPES OF EVENTS

*MUTUALLY EXCLUSIVE*: TWO (OR MORE) EVENTS THAT CANNOT OCCUR AT THE SAME TIME

EVENT A: VERY LOW BIRTH WEIGHT (<1500 GRAMS)

EVENT B: LOW BIRTH WEIGHT (1500 - <2500 GRAMS)

SAMPLE SPACE: {4,119 BIRTHS AT TWO HOSPITALS}

EVENT A: 138 BIRTHS <1500 GRAMS

EVENT B: 351 BIRTHS 1500 - <2500 GRAMS

WHAT IS THE PROBABILITY OF A BIRTH <2500 GRAMS?

$$P(A \text{ OR } B) = P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = (138 / 4119) + (351 / 4119)$$

$$P(A \cup B) = 0.0335 + 0.0852 = 0.119$$

*INDEPENDENT*: THE OCCURRENCE OF ONE EVENT DOES NOT AFFECT THE PROBABILITY OF OCCURRENCE OF ANOTHER EVENT (OR OTHER EVENTS)

EVENT A: ROLL ONE DICE

EVENT B: ROLL ONE DICE (AGAIN)

EVENT A: BIRTH AND INFANT IS LOW BIRTH WEIGHT

EVENT B: ANOTHER BIRTH AND INFANT IS LOW BIRTH WEIGHT

GIVEN TWO ROLLS OF A DICE, WHAT IS THE PROBABILITY OF ROLLING A 3 FOLLOWED BY ANOTHER 3?

YOU KNOW THAT...

1 OUT OF 6 SIDES OF A DICE IS A 3

$$P(A \text{ AND } B) = P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = (1 / 6) \times (1 / 6)$$

$$P(A \cap B) = 0.167 \times 0.167 = 0.0278$$

OR...WHAT IS THE SAMPLE SPACE?

{1/1, 1/2, 1/3, 1/4, 1/5, 1/6,  
2/1, 2/2, 2/3, 2/4, 2/5, 2/6,  
3/1, 3/2, 3/3, 3/4, 3/5, 3/6,  
4/1, 4/2, 4/3, 4/4, 4/5, 4/6,  
5/1, 5/2, 5/3, 5/4, 5/5, 5/6,  
6/1, 6/2, 6/3, 6/4, 6/5, 6/6}

36 POSSIBLE OUTCOMES, ALL WITH SAME PROBABILITY  
1 EVENT - TWO CONSECUTIVE 3'S

$$P(A) = 1 / 36 = 0.0278$$

GIVEN TWO BIRTHS, WHAT IS THE PROBABILITY THAT THEY ARE BOTH LOW BIRTH WEIGHT INFANTS?

YOU KNOW THAT ...

489 OUT OF 4,119 BIRTHS WERE < 2500 GRAMS

$$P(A \text{ AND } B) = P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = (489 / 4119) \times (489 / 4119)$$

$$P(A \cap B) = 0.119 \times 0.119 = 0.014$$

OR...GIVEN TWO BIRTHS, WHAT IS THE PROBABILITY THAT AT LEAST ONE IS A LOW BIRTH WEIGHT INFANTS?

OUT OF 4119 BIRTHS, 489 LOW, 3630 NORMAL

LET  $P(\bar{A})$  = PROBABILITY OF NO LOW BIRTH WEIGHT INFANTS

$$P(\bar{A}) = (3630 / 4119) * (3630 / 4119)$$

$$P(\bar{A}) = 0.881 * 0.881 = 0.776$$

LET  $P(A)$  = PROBABILITY OF AT LEAST ONE LOW

$$P(A) = 1 - P(\bar{A}) = 1 - 0.776 = 0.224$$

ANOTHER APPROACH...

WHAT IS THE SAMPLE SPACE...

{NORMAL/NORMAL	$0.881 \times 0.881 = 0.776$
NORMAL/LOW	$0.881 \times 0.119 = 0.105$
LOW/NORMAL	$0.119 \times 0.881 = 0.105$
LOW/LOW}	$0.119 \times 0.119 = 0.014$

4 POSSIBLE OUTCOMES THAT DO NOT ALL HAVE THE SAME PROBABILITY

AT LEAST ONE LOW BIRTH WEIGHT INFANT...

$$P(A) = 0.105 + 0.105 + 0.014$$

$$P(A) = .224$$

GENERAL RULE FOR SIZE OF SAMPLE SPACE...NUMBER OF POSSIBLE OUTCOMES...

$2^N$  WHERE N IS THE NUMBER OF TRIALS

TRIALS	OUTCOMES
1	2
2	4
3	8
4	16
5	32
.	
.	
.	
10	1024

OR...GIVEN TEN BIRTHS, WHAT IS THE PROBABILITY THAT AT LEAST ONE IS A LOW BIRTH WEIGHT INFANTS?

SIZE OF SAMPLE SPACE =  $2^{10} = 1024$

THINK NEGATIVE...ONLY ONE EVENT WITH NO LOW BIRTH WEIGHT INFANTS...ALL NORMAL WEIGHT...

$$P(A) = 1 - P(\bar{A})$$

$$P(A) = 1 - 0.881^{10} = 1 - 0.282 = 0.718$$

## PROBABILITY RULES

*ADDITION RULE*: MUTUALLY EXCLUSIVE (SOMETIMES CALLED 'DISJOINT')...

$$P(A \text{ OR } B) = P(A \cup B) = P(A) + P(B)$$

$$P(A \text{ OR } B \text{ OR } C) = P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

*ADDITION RULE*: EVENTS *NOT* MUTUALLY EXCLUSIVE...

$$P(\text{OR } B) = P(A \cup B)$$

$$P(\text{OR } B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \text{ OR } B \text{ OR } C) = P(A \cup B \cup C)$$

$$\begin{aligned} P(A \text{ OR } B \text{ OR } C) = & P(A) + P(B) + P(C) \\ & - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ & + P(A \cap B \cap C) \end{aligned}$$

FROM ROSNER STUDY GUIDE, CHAPTER 3...EAR INFECTIONS...

PROBABILITY ONE EAR INFECTED: 0.10

PROBABILITY BOTH EARS INFECTED: 0.07

HYPOTHETICAL 100 CHILDREN

HOW WOULD YOU CONSTRUCT THIS TABLE?

		LEFT EAR INFECTED		
		YES	NO	TOTAL
RIGHT EAR INFECTED	YES	7	3	10
	NO	3	87	90
	TOTAL	10	90	100

*NOT MUTUALLY EXCLUSIVE SINCE  $P(A \cap B) \neq 0$*

WHAT IS THE PROBABILITY THAT *EITHER* EAR IS INFECTED?

*ROSNER ALWAYS USES A AND B, SO...*

$P(A) = P(R)$ : RIGHT EAR INFECTED

$P(B) = P(L)$ : LEFT EAR INFECTED

$$P(R \text{ OR } L) = P(R \cup L) = P(R) + P(L) - P(R \cap L)$$

$$P(R \cup L) = (10 / 100) + (10 / 100) - (7 / 100)$$

$$P(R \cup L) = 0.10 + 0.10 - 0.07 = 0.13$$

OR...LOOKING AT THE 2 x 2 TABLE...

$$P(R \cup L) = (7 + 3 + 3) / 100$$

$$P(R \cup L) = 13 / 100 = 0.13$$

COULD YOU SOLVE THIS USING A SAMPLE SPACE?

SAMPLE SPACE APPROACH ASSUMES INDEPENDENCE

*MULTIPLICATION RULE*: INDEPENDENT EVENTS...

$$P(A \text{ AND } B) = P(A \cap B) = P(A) \times P(B)$$

$$P(A \text{ AND } B \text{ AND } C) = P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

*MULTIPLICATION RULE*: NON-INDEPENDENT EVENTS...

$$P(A \text{ AND } B) = P(A \cap B) \neq P(A) \times P(B)$$

$$P(A \text{ AND } B) = P(A \cap B) = P(A) \times P(B | A)$$

OR...

$$P(A \text{ AND } B) = P(A \cap B) = P(A|B) \times P(B)$$

$P(B | A)$  IS *CONDITIONAL PROBABILITY*

$$P(B | A) = P(A \cap B) / P(A)$$

WHAT IS THE PROBABILITY OF EVENT B GIVEN THAT EVENT A HAS ALREADY OCCURRED

$P(A | B)$  IS *CONDITIONAL PROBABILITY*

$$P(A | B) = P(A \cap B) / P(B)$$

WHAT IS THE PROBABILITY OF EVENT A GIVEN THAT EVENT B HAS ALREADY OCCURRED

ARE LEFT-EAR AND RIGHT-EAR INFECTIONS INDEPENDENT?

IF INDEPENDENT ... HYPOTHETICAL 100 CHILDREN

$$P(R \text{ AND } L) = P(R \cap L) = P(R) \times P(L)$$

$$P(R \cap L) = 0.10 \times 0.10 = 0.01$$

HOW WOULD YOU CONSTRUCT THIS TABLE?

		LEFT EAR INFECTED		
		YES	NO	TOTAL
RIGHT EAR INFECTED	YES	1	9	10
	NO	9	81	90
	TOTAL	10	90	100

OR...LOOKING AT THE 2 x 2 TABLE...

$$P(R \cap L) = 1 / 100$$

$$P(R \cap L) = 0.01$$

HOWEVER..

WE ARE TOLD THAT  $P(R \cap L) = 0.07$

THEREFORE...

*NOT INDEPENDENT*

HYPOTHETICAL 100 CHILDREN

		LEFT EAR INFECTED		
		YES	NO	TOTAL
RIGHT EAR INFECTED	YES	7	3	10
	NO	3	87	90
	TOTAL	10	90	100

CONDITIONAL PROBABILITY...

$$P(R \text{ AND } L) = P(R \cap L) = P(R) \times P(L | R)$$

THEREFORE...

$$P(L | R) = P(R \cap L) / P(R)$$

$$P(L | R) = (7/100) / (10/100)$$

$$P(L | R) = 0.07 / 0.10 = 0.70$$

OR...LOOKING AT THE 2 x 2 TABLE...THE PROBABILITY THAT THE LEFT EAR IS INFECTED GIVEN THAT THE RIGHT EAR IS INFECTED IS THE FIRST ROW OF THE TABLE...

$$P(L | R) = 7/10 = 0.70$$

THE PROBABILITY THAT THE LEFT EAR IS INFECTED GIVEN THAT THE RIGHT EAR IS *NOT* INFECTED IS THE SECOND ROW OF THE TABLE...

$$P(L | \bar{R}) = 3/90 = 0.03$$

RELATIVE RISK: WHAT IS THE RISK OF EVENT B GIVEN THAT EVENT A HAS OCCURRED *RELATIVE* TO THE RISK OF EVENT B GIVEN THAT EVENT A HAS NOT OCCURRED

$$RR = P(L | R) / P(L | \bar{R})$$

$$RR = 0.70 / 0.03 \approx 21$$

OR...AN INFANT WITH AN INFECTION IN THE RIGHT EAR IS 21 TIMES AS LIKELY TO HAVE AN INFECTION IN THE LEFT EAR AS AN INFANT WITH NO INFECTION IN THE RIGHT EAR

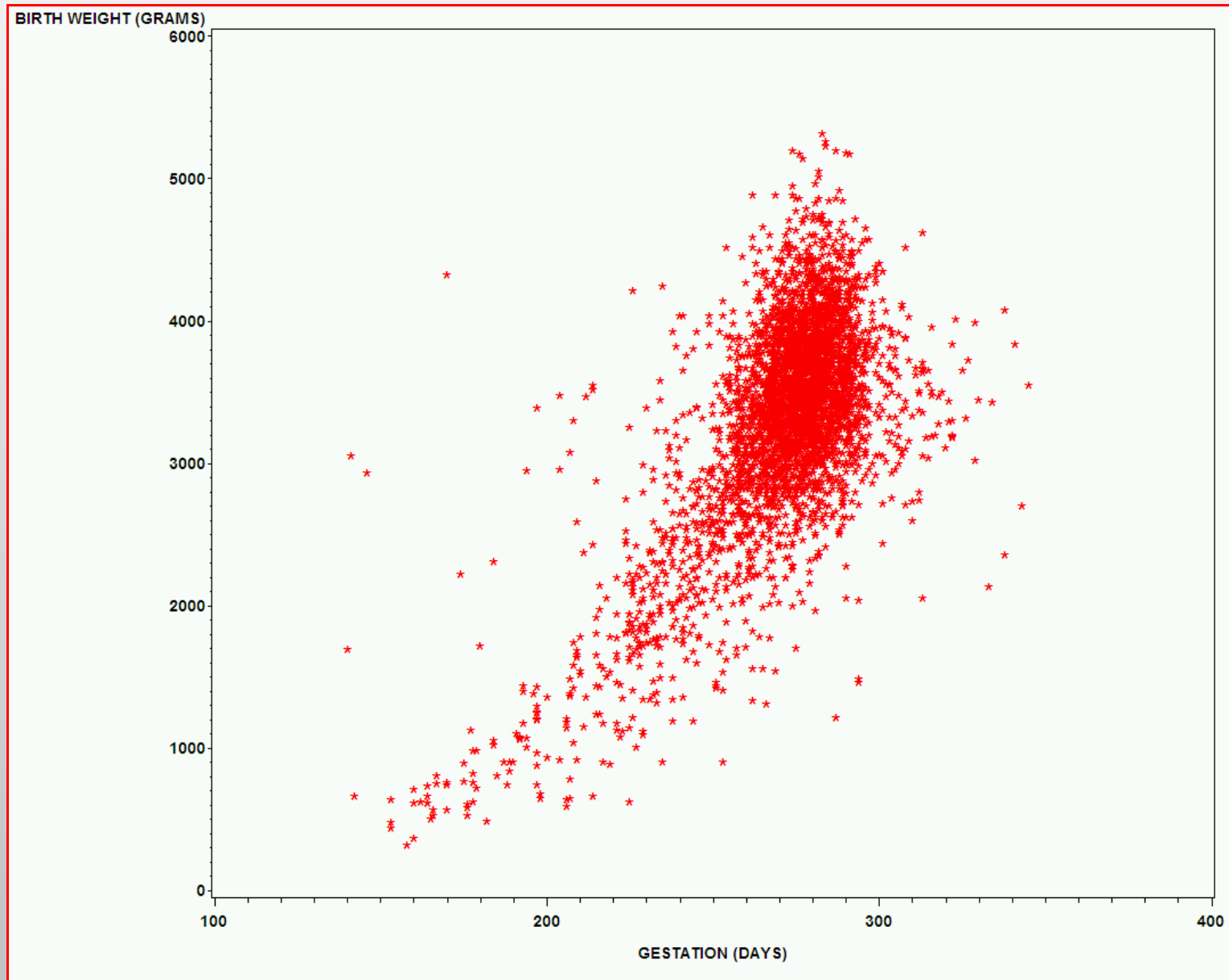
ANOTHER EXAMPLE...BIRTH DATA FROM TWO HOSPITALS...

WHAT IS THE RISK OF HAVING A LOW BIRTH WEIGHT INFANT GIVEN SHORT GESTATION RELATIVE TO THAT OF NORMAL GESTATION?

LOW BIRTH WEIGHT:  $< 2500$  GRAMS

SHORT GESTATION:  $< 37$  WEEKS ( $< 259$  DAYS)

IS BIRTH WEIGHT RELATED TO GESTATION?  
SCATTER PLOT OF ALL BIRTHS AT TWO HOSPITALS...



4,048 BIRTHS WITH 'GOOD DATA' ON BOTH BIRTH WEIGHT AND GESTATION...

		BIRTH WEIGHT		
		LOW	NORMAL	TOTAL
GESTATION	SHORT	367	270	637
	NORMAL	103	3308	3411
	TOTAL	470	3578	4048

ARE GESTATION AND BIRTH WEIGHT INDEPENDENT?

P(G): SHORT GESTATION

P(B): LOW BIRTH WEIGHT

$$P(G \cap B) = P(G) \times P(B)$$

$$P(G \cap B) = (637 / 4048) \times (470 / 4048)$$

$$P(G \cap B) = 0.157 \times 0.116 = 0.018$$

BUT...FROM THE 2 x 2 TABLE...

$$P(G \cap B) = 367 / 4048 = 0.091$$

*BIRTH WEIGHT AND GESTATION ARE NOT INDEPENDENT*

THEREFORE...

$$P(B | G) = P(G \cap B) / P(G)$$

$$P(B | G) = (367 / 4048) / (637 / 4048)$$

$$P(B | G) = 0.091 / 0.157 = 0.576$$

AND...

$$P(B | \bar{G}) = P(B \cap \bar{G}) / P(\bar{G})$$

$$P(B | \bar{G}) = (103 / 4048) / (3411 / 4048)$$

$$P(B | \bar{G}) = 0.025 / 0.843 = 0.030$$

$$RR = P(B | G) / P(B | \bar{G})$$

$$RR = 0.576 / 0.030 = 19.3$$

OR...AN INFANT WITH SHORT GESTATION IS 19 TIMES AS LIKELY TO BE LOW BIRTH WEIGHT THAN ONE WITH NORMAL GESTATION

OR ... USING THE TABLE RATHER THAN FORMULAS...

FROM ROW 1 ... CONDITIONED ON SHORT GESTATION

$$P(B | G) = 367 / 637 = 0.576$$

FROM ROW 2 ... CONDITIONED ON NORMAL GESTATION

$$P(B | \bar{G}) = 103 / 3411 = 0.030$$

$$RR = P(B | G) / P(B | \bar{G})$$

$$RR = 0.576 / 0.030 = 19.3$$

## **SENSITIVITY, SPECIFICITY, PREDICTIVE VALUE**

### **KNOWN DISEASE STATUS...**

*SENSITIVITY*: PROBABILITY OF A TRUE POSITIVE GIVEN DISEASE

*SPECIFICITY*: PROBABILITY OF A TRUE NEGATIVE GIVEN NO DISEASE

### **KNOWN TEST RESULT...**

*PREDICTIVE VALUE OF A POSITIVE TEST*: PROBABILITY OF A TRUE POSITIVE GIVEN A POSITIVE TEST RESULT

*PREDICTIVE VALUE OF A NEGATIVE TEST*: PROBABILITY OF A TRUE NEGATIVE GIVEN A NEGATIVE TEST RESULT

***ALL ARE CONDITIONAL PROBABILITIES***

FROM GALEN AND GAMBINO...

		TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
REALITY	DISEASE	TP	FN	TP+FN
	NO DISEASE	FP	TN	FP+TN
	TOTAL	TP+FP	FN+TN	TOTAL

TP: TRUE POSITIVE

FP: FALSE POSITIVE

FN: FALSE NEGATIVES

TN: TRUE NEGATIVES

FROM 2x2 TABLE...

$$\text{SENSITIVITY} = \text{TP} / (\text{TP} + \text{FN})$$

$$\text{SPECIFICITY} = \text{TN} / (\text{FP} + \text{TN})$$

$$\text{PREDICTIVE VALUE OF A + TEST} = \text{TP} / (\text{TP} + \text{FP})$$

$$\text{PREDICTIVE VALUE OF A - TEST} = \text{TN} / (\text{FN} + \text{TN})$$

FROM TRIOLA...

		TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
PREGNANT	YES	80	5	85
	NO	3	11	14
	TOTAL	83	16	99

$$\text{SENSITIVITY} = 80 / 85 = 0.941$$

$$\text{SPECIFICITY} = 11 / 14 = 0.786$$

$$\text{PREDICTIVE VALUE + TEST} = 80 / 83 = 0.964$$

$$\text{PREDICTIVE VALUE - TEST} = 11 / 16 = 0.687$$

WHAT DOES THIS SAY TO YOU?

FROM CARTOON GUIDE...

TWO EVENTS: A - PATIENT HAS THE DISEASE  
B - PATIENT TESTS POSITIVE

INFORMATION:

$P(A) = .001$  (1 PATIENT IN 1,000 HAS DISEASE)

$P(B | A) = .990$  (P + TEST GIVEN DISEASE)

$P(B | \bar{A}) = .020$  (P FALSE + GIVEN NO DISEASE)

QUESTION...WHAT IS THE PROBABILITY OF HAVING THE DISEASE GIVEN A POSITIVE TEST... $P(A | B)$  ?

CONSTRUCT A 2 x 2 TABLE...  
 HYPOTHETICAL 100,000 PEOPLE

		TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
DISEASE	YES	99	1	100
	NO	1,998	97,902	99,900
	TOTAL	2,097	97,903	100,000

SENSITIVITY =  $99 / 100 = 0.990$

SPECIFICITY =  $97902 / 99900 = 0.980$

PREDICTIVE VALUE OF A + TEST =  $99 / 2097 = 0.047$

OR...

LESS THAN 5% OF THOSE WHO TEST + HAVE THE DISEASE

WHAT DOES THIS SAY TO YOU?

NO TEST: 1 / 1000 CHANCE OF HAVING DISEASE

+ TEST: 1 / 21 CHANCE OF HAVING DISEASE

ENOUGH EVIDENCE TO START A TREATMENT?

CONSIDERATIONS...

CONSEQUENCES OF HAVING THE DISEASE

CONSEQUENCES OF THE TREATMENT

CHANGE PREVALENCE TO 10 PER 1,000

CONSTRUCT A 2 x 2 TABLE...

		TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
DISEASE	YES	990	10	1,000
	NO	1,980	97,020	99,000
	TOTAL	2,970	97,030	100,000

SENSITIVITY AND SPECIFICITY STAY THE SAME

PREDICTIVE VALUE OF A + TEST =  $990 / 2970 = 0.333$

OR...

33% OF THOSE WHO TEST + HAVE THE DISEASE

WHAT DOES THIS SAY TO YOU?

FROM GALEN AND GAMBINO...COLON CANCER EXAMPLE

EFFECT OF PREVALENCE ON...

PREDICTIVE VALUES POSITIVE TEST

PREDICTIVE VALUE OF NEGATIVE TESTS

EFFICIENCY (NEW TERM BASED ON BOTH OF THE ABOVE)

PREVALENCE: 1,000 PER 100,000 (0.01)

SENSITIVITY:  $720 / 1000 = 0.72$

SPECIFICITY:  $79200 / 99000 = 0.80$

		TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
COLON CANCER	YES	720	280	1,000
	NO	19,800	79,200	99,000
	TOTAL	20,520	79,480	100,000

PREDICTIVE VALUE + TEST:  $720 / 20520 = 0.035$

PREDICTIVE VALUE - TEST:  $79200 / 79480 = 0.996$

EFFICIENCY:  $(720 + 79200) / 100000 = 0.799$  (79.9%)

PREVALENCE: 10,000 PER 100,000 (0.10)

SENSITIVITY:  $7200 / 10000 = 0.72$

SPECIFICITY:  $72000 / 90000 = 0.80$

		TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
COLON CANCER	YES	7,200	2,800	10,000
	NO	18,000	72,000	90,000
	TOTAL	25,200	74,800	100,000

PREDICTIVE VALUE + TEST:  $7200 / 25200 = 0.286$

PREDICTIVE VALUE - TEST:  $72000 / 74800 = 0.963$

EFFICIENCY:  $(7200 + 72000) / 100000 = 0.792$  (79.2%)

PREVALENCE: 50,000 PER 100,000 (0.50)

SENSITIVITY:  $36000 / 50000 = 0.72$

SPECIFICITY:  $40000 / 50000 = 0.80$

		TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
COLON CANCER	YES	36,000	14,000	50,000
	NO	10,000	40,000	50,000
	TOTAL	46,000	54,000	100,000

PREDICTIVE VALUE + TEST:  $36000 / 46000 = 0.783$

PREDICTIVE VALUE - TEST:  $40000 / 54000 = 0.740$

EFFICIENCY:  $(36000 + 40000) / 100000 = 0.760$  (76.0%)

SUMMARY...

SENSITIVITY: 0.72

SPECIFICITY: 0.80

PREVALENCE	PREDICTIVE VALUE +	PREDICTIVE VALUE -	EFFICIENCY
0.010	3.5%	99.6%	79.9%
0.100	28.6%	96.3%	79.2%
0.500	78.3%	74.0%	76.0%

WHAT IS THE EFFECT OF PREVALENCE ON PREDICTIVE VALUES?

**\*\*\*\*\* EXTRA MATERIAL \*\*\*\*\***

## CONDITIONAL PROBABILITY

EXAMPLE 3.15 FROM ROSNER ... SYPHILIS DIAGNOSIS ...

$P(A)$  = POSITIVE DIAGNOSIS DOCTOR A = 0.10

$P(B)$  = POSITIVE DIAGNOSIS DOCTOR B = 0.17

FROM FREQUENCY DEFINITION OF PROBABILITY SINCE YOU ARE TOLD THAT DOCTOR A MAKES A + DIAGNOSIS IN 10% OF PATIENTS, WHILE DOCTOR B DOES SO IN 17%

HYPOTHETICAL 1000 PATIENTS ... IF TEST RESULTS INDEPENDENT ...

		DOCTOR A		
		POSITIVE	NEGATIVE	TOTAL
DOCTOR B	POSITIVE	17	153	170
	NEGATIVE	83	747	830
	TOTAL	100	900	1000

WHERE DO THE NUMBERS COME FROM? YOU ARE TOLD THAT BOTH DOCTORS ARE HAVE + RESULTS ON 8% OF PATIENTS, SO ...

		DOCTOR A		
		POSITIVE	NEGATIVE	TOTAL
DOCTOR B	POSITIVE	80	90	170
	NEGATIVE	20	810	830
	TOTAL	100	900	1000

WHERE DO THE NUMBERS COME FROM? ARE THE RESULTS FROM DOCTOR A AND DOCTOR B INDEPENDENT? WHY OR WHY NOT?

CONDITIONAL PROBABILITY QUESTIONS ... BOTH TABLES

IF DOCTOR A MAKES A POSITIVE DIAGNOSIS, WHAT IS THE PROBABILITY THAT DOCTOR B MAKES A POSITIVE DIAGNOSIS?

IF DOCTOR B MAKES A POSITIVE DIAGNOSIS, WHAT IS THE PROBABILITY THAT DOCTOR A MAKES A POSITIVE DIAGNOSIS?

## TOTAL PROBABILITY

EXAMPLE 3.21 IN ROSNER ... BREAST CANCER/MAMMOGRAM ...

PROBABILITY OF BREAST CANCER WITHIN 2 YEARS GIVEN A NEGATIVE MAMMOGRAM IS 0.0002

PROBABILITY OF BREAST CANCER WITHIN 2 YEARS GIVEN A POSITIVE MAMMOGRAM IS 0.1

7% OF WOMEN IN GENERAL POPULATION HAVE A POSITIVE MAMMOGRAM

WHAT IS THE PROBABILITY OF DEVELOPING BREAST CANCER WITHIN 2 YEARS IN THE GENERAL POPULATION?

HYPOTHETICAL 1,000,000 WOMEN

		MAMMOGRAM RESULT		
		POSITIVE	NEGATIVE	TOTAL
BREAST CANCER	YES	7,000	186	7,186
	NO	63,000	929,814	992,814
	TOTAL	70,000	930,000	1,000,000

WHERE DO THE NUMBERS COME FROM?

$$P(\text{BREAST CANCER}) = 7,186 / 1,000,000 = 0.00719$$

## SCREENING TESTS

REVIEW QUESTIONS 3C IN ROSNER

PSA TEST RESULT	PROSTATE CANCER	FREQUENCY
+	+	92
+	-	27
-	+	46
-	-	72

WHAT ARE SENSITIVITY, SPECIFICITY, PREDICTIVE VALUES?

		PSA TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
PROSTATE CANCER	YES	92	46	138
	NO	27	72	99
	TOTAL	119	118	237

SENSITIVITY:  $92 / 138 = 0.667$

SPECIFICITY:  $72 / 99 = 0.727$

PREDICTIVE VALUES ...

POSITIVE TEST:  $92 / 119 = 0.773$

NEGATIVE TEST:  $72 / 118 = 0.610$

EXTRA...

IF PREVALENCE IN GENERAL POPULATION IS 20%,  
WHAT ARE THE PREDICTIVE VALUES?

HYPOTHETICAL 10,000 MEN

		PSA TEST RESULT		
		POSITIVE	NEGATIVE	TOTAL
PROSTATE CANCER	YES	1,334	666	2,000
	NO	2,184	5,816	8,000
	TOTAL	3,518	6,484	10,000

WHERE DO THE NUMBERS COME FROM?

PREDICTIVE VALUES ...

POSITIVE TEST:  $1,334 / 3,518 = 0.379$

NEGATIVE TEST:  $5,816 / 6,484 = 0.897$