Problem Answers Chapter 8

8.1

Ho: $\mu = 0$
H1: $\mu \neq 0$

This is a paired t-test with a null hypothesis that the mean change = 0 versus the alternative that the mean change does not = 0 (two sided alternative). Using equation 8.4 on page 299 of Rosner with the values of 20.0 for the sample mean change, 35.0 for the sample standard deviation, and 20 for the sample size, t = 2.556. At the 5% level, the critical values are +2.093 and -2.093 (alpha/2 = .025, df = 19). Since 2.556 is greater than 2.093, we reject the null hypothesis at the 5% level.

8.2

In this case, the calculated t value is 1.118 using equation 8.4. The null and alternative hypotheses remain the same. The critical values here are the same as in problem 8.1 since df = 19 and alpha/2 = .025. Here since the calculated value for t does not exceed the critical value, we accept the null hypothesis at the 5% level.

8.3

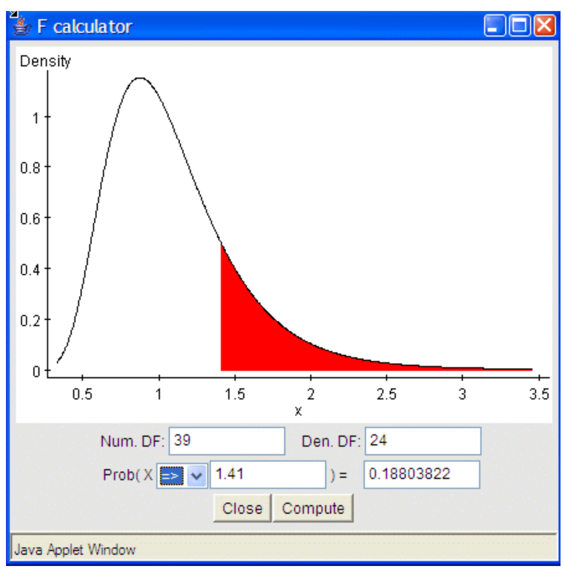
Here we again use the same methods. The calculated t value is 2.907 (mean change = 5.2, mean standard deviation = 8.0, sample size = 20) using equation 8.4. Use the same critical values since the df an alpha level are the same. Since 2.907 exceeds 2.093, we reject the null hypothesis at the 5% level.

8.4

Use Equation 8.14 on page 313 since you want to find the lower 2.5th percentile. The lower 2.5th percentile of an F distribution with 14 df in the numerator and 7 df in the denominator is equal to the reciprocal of the upper 2.5th percentile of an F distribution with 7 df in the numerator and 14 df in the denominator. Using Table 9 in Rosner with 7 df in the numerator, 14 df in the denominator and p = .975, we find F = 3.38. So, F for df = 14 in the numerator, df = 7 in the denominator and p = .025 is 1 divided by 3.38 = 0.296.

8.5

To test the null hypothesis that the two variances are equal versus the 2 sided alternative hypothesis that they are not equal use the F test. F = the ratio of the two variances (equation 8.15 on page 313 of Rosner). In this case F = .76 squared / .64 squared = 1.410. The critical value for F with 39 df in the numerator and 24 df in the denominator is not given exactly in Table 9 or Rosner.



You can use StatCrunch to calculate the exact value.

But even without doing that, if you look at Table 9 you will see that since the calculated value for F is less than the critical value of df = infinity in the numerator and 30 in the denominator with p = .975, it will also be less than the critical value we really want to compare to. So, we conclude there is not significant difference in the variances at the 5% level.

8.6

Since we accepted the null hypothesis in Problem 8.5, we want to use the two sample t-test for independent samples with equal variances.

8.7

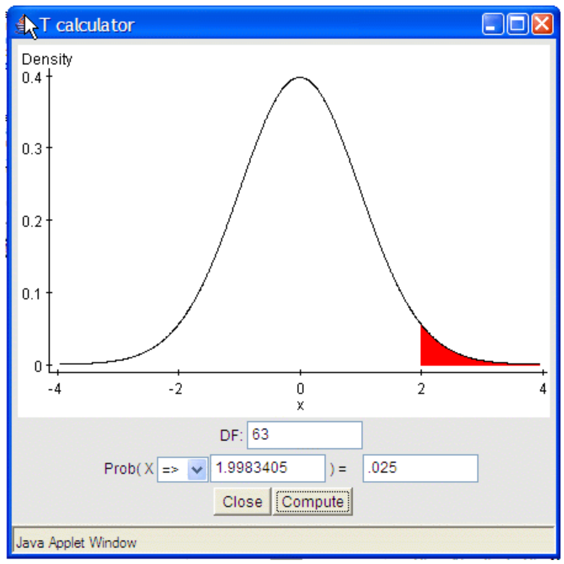
Ho: µ1 = µ2
H1: µ1 ≠ µ2

First calculate the pooled variance estimate, using equation 8.10 on Page 305 of Rosner. The calculated value of s squared is 32.357/63 = 0.514. Now compute the test statistic t using equation 8.11 on Page 305 of Rosner. The calculated value for t is -.024 / 0.183 = -1.314. The critical value for t with 63 df (40+25-2) and p = .975 is going to be somewhere between 1.980 and 2.000 if you look at Table 5 of Rosner. Since the calculated t value (the absolute value) is less than the critical value, we accept the null hypothesis at the 5% level.

8.8

Use the method outlined in equation 8.12 on Page 306 of Rosner. The exact value of t is not given in Table 5 since there is no entry for df=63. But you can determine an estimate using the closest values for degrees of freedom available. Using 60 degrees of freedom, the p value is between .10 and .20. Likewise, for 120 degrees of freedom, the p value is between .10 and .20. Thus, since we have 63 degrees of freedom, it must be between .10 and.20.

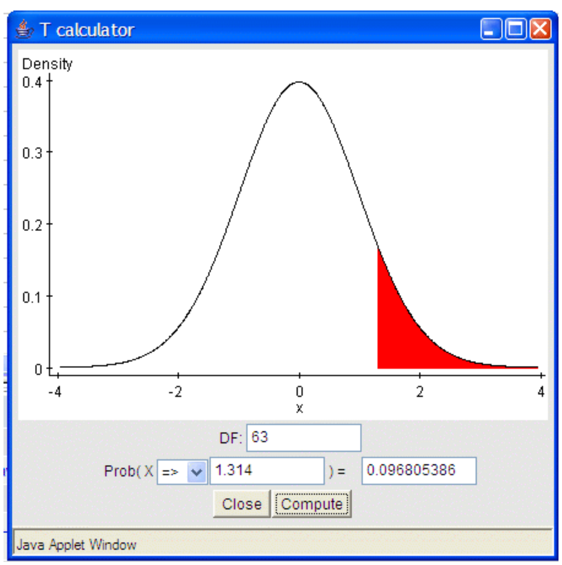
Using StatCrunch, the p value can be calculated exactly (remember StatCrunch is 1-tail) .



8.9

Use equation 8.13 on Page 309 of Rosner to calculate the confidence interval. Using StatCrunch, you can find the critical value for t with 63 degrees of freedom and p = .05 is 1.9983. The confidence interval is ...

-.025 +/- 1.9983(0.183) = (-0.61, 0.13).



8.10

There are 6 people who received a bacterial culture with mean WBC of 9.50 and standard deviation of 3.39. There are 19 people who did not receive a bacterial culture and their mean WBC is 7.32 with a standard deviation of 3.06. Assuming that the distribution of WBC is normal in both groups, we can test whether or not the two variances are equal using equation 8.15 on Page 313 of Rosner. F is calculated by taking the ratio of the two variances and is 1.23. The critical value for an F distribution with 5 degrees of freedom in the numerator and 18 degrees of freedom in the denominator with p = .975 is given in Table 9 of Rosner is 3.38. Since our calculated F is less than the critical value, the variances are not significantly different at the 5% level.

8.11

Based upon the result in Problem 8.10, the appropriate test procedure is the two sample t-test for independent samples with equal variances.

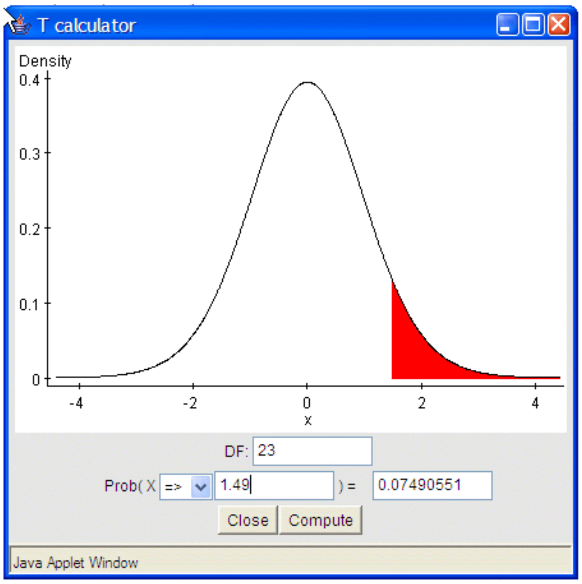
8.12

Ho: $\mu1 = \mu2$
H1: $\mu1 \neq \mu2$

This problem is approached exactly like Problem 8.7. Using equation 8.10 to calculate the pooled variance and equation 8.11 to calculate the t value, you get t = 1.49. The critical value for t with 23 df and p = .975 (from Table 5 in Rosner) is 2.069. Since the calculated value is less than the critical value, we accept the null hypothesis at the 5% level and conclude that the means are equal.

8.13

Using Table 5 in Rosner, with df=23, you can estimate that the p value is between .10 and .20. Using StatCrunch, you can calculate the exact p value (remember StatCrunch is 1-tail) .



8.14

The confidence interval is calculated using equation 8.13 on Page 309 of Rosner ...

2.18 +/- 2.069(1.467) = 2.18 +/- 3.03 = (-0.85, 5.22).

8.153

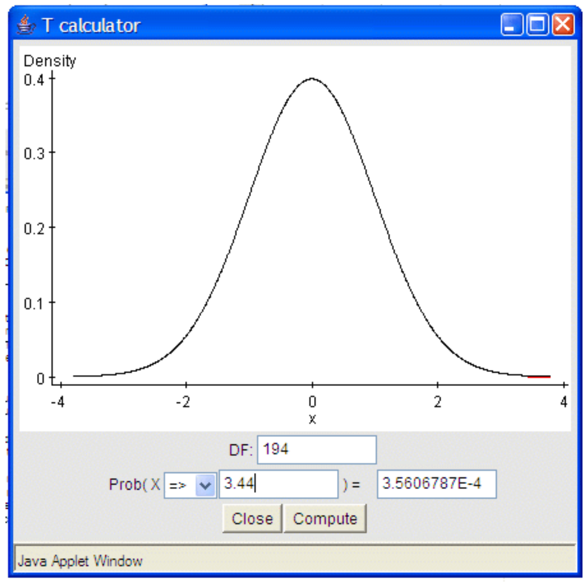
We use a two sample t-test to compare the cases and controls. We can check for equality of variance using the information given in the table: F = 28.8 squared / 27.5 squared = 1.097 with 104,90 degrees of freedom. The critical value at the .05 level is given as 1.498. Therefore, the variances are equal and we can use the two sample t-test with equal variances.

8.154

Ho: $\mu1 = \mu2$
H1: $\mu1 \neq \mu2$

The pooled variance (using equation 8.10) is 795.468. Using equation 8.11, calculate t = -3.44 (with mean1 as the control mean, and mean2 as the case mean). There is no value for t with 194 degrees of freedom in table 5 in the appendix. However, all the values for t with 120 degrees of freedom in the last four columns of the table are lower than absolute value of t calculated using study data, 3.44 (even in the .9995 column). Given that tabulated values of t with 194 degrees of freedom would be lower than those at 120 degrees of freedom, we conclude that we can reject the null hypothesis of equality of means --- there is a significant difference at a <.001 level.

Using StatCrunch to calculate an exact p value (remember StatCrunch is 1-tail, and for those of you who do not recognize it, that is scientific notation for the value 0.00035606787).



8.155

Using equation 8.13 and the value given for t 194,.975, a 95% confidence interval for the true mean difference is (-21.9, -5.9). Notice that the confidence band does not include 0, leading to the same conclusion as the t-test in problem 8.154, that the mean difference is not zero.

8.156

A 99% confidence interval band would be wider than a 95% confidence interval since if you want to state that "... on repeated sampling the true difference would lie between the confidence limits 99% of the time instead of 95% ... " you need wider limits.

What limits would give you a 100% confidence ... infinitely wide ... they are also wider than 95%.