Price Dynamics and the Market for Access to Trading Partners*

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Abstract

At each point in time, price dynamics in a market are determined by a market for access to trading partners, implemented by competitive profit-maximizing brokers. This mechanism is applied to a market in which the value of a good declines over time and buyers decide optimally when to reenter the market and buy a new unit. Price adjustment paths in response to increases and decreases in demand are then derived using the differential equations generated by the model.

1. Introduction

This paper demonstrates that equilibrium in a market for access to trading partners can be used to analyze disequilibrium price dynamics in a market for a good. The market for access to trading partners is generated by competitive profit-maximizing brokers who charge fees and can offer alternative ratios of buyers to

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sellers in a market in which the number of trades is determined by a matching function. At each point in time, the fee charged by brokers determines the change in the price, which yields the disequilibrium price dynamics.

In the standard textbook supply and demand model without frictions, there is no cost to finding a trading partner in equilibrium. In disequilibrium, with excess demand or supply, some buyers or sellers are unable to find trading partners at the current price. With excess demand, for example, not all buyers are able to find sellers, so that access to trading partners has a value to buyers. However, that access is not priced in the standard model. A market for access to trading partners, if it operated, would set a fee for access to sellers. This fee, added to the current price, would determine the disequilibrium price. If the price is updated at each point in time, the fee determines the rate of change of the price.

In this paper, the market for access to trading partners is implemented by profit-seeking brokers. At any point in time, the number of trades is a function (called the matching function) of the number of buyers and number of sellers.[1] Buyers prefer a lower ratio of buyers to sellers, and are willing to pay a fee to get a ratio lower than a given ratio. Sellers prefer a higher ratio of buyers to sellers, and would need to be paid a fee to accept a lower ratio. Buyers and sellers both have trade-offs between the fee and the ratio of buyers to sellers. Competition among brokers results in fees such that buyer and seller indifference curves are tangent. A market for access to trading partners then arises because buyers and sellers face the same trade-offs between fee and ratio of buyers to sellers, which affects the likelihood of trading. The market for access to trading partners implemented by brokers determines the fee and price change in the short run, in which some element of the market is in disequilibrium. The role of brokers in bringing about equilibrium in a market is discussed in Sattinger (2000). Related work on brokers (or middlemen) and equilibrium is by Nti and Shubik (1984), Rubenstein and Wolinsky (1987), Gehrig (1993), Yavaş (1994, 1996), Serrano (1995), Wooders (1997) and Mortensen and Wright (1997).

Characterization of price determination through a market for access to trading partners has several antecedents. Search is a market mechanism used by agents to find trading partners (see the survey by Mortensen and Pissarides, 1999). In the context of labor markets, Sattinger (1990) analyzes a market for interviews that generates the same efficiency conditions as occur in this paper (see also Sattinger, 1995, for a comparison with Nash bargaining). Moen (1997) develops

[1] The matching function is discussed by Mortensen and Pissarides (1999, pp. 2575-2578) and Pissarides (2000, pp. 6-7).

a model of competitive search equilibrium. Various authors have studied markets for contracts, including Gale (1992, 1994), Peters (1997, 1999), Shimer (1996) and Wilson (1989). Chatterjee and Dutta (1998) analyze competition for bargaining partners. Sattinger (2002) analyzes a queuing mechanism that generates a market for access to trading partners. Lu and McAfee (1996) and Moreno and Wooders (1999) have analyzed price dynamics with rationing.

The basic mechanism for determining the price in disequilibrium must be combined with specific assumptions about the market for the good in order to generate deterministic price paths. In this paper, buyers purchase a single unit of a durable good that declines in value over time. At an optimally determined age of the good, they dispose of the good and reenter the market to buy a new one. If the terms of trade improve (e.g., the price goes down), buyers will reenter the market sooner, so that the number of buyers in the market at one point in time goes up. Sellers face a particular cost of the good and reenter the market as soon as they have sold the good. At a particular point in time, buyers are heterogeneous because they hold goods acquired at different times in the past. However, when they enter the market to buy a new good, their valuation of the good is the same. As a result of these assumptions, buyers on the market at a point in time will have identical indifference curves through a point (determined by the fee, prevailing price, and ratio of buyers to sellers). If buyers and sellers in the market were heterogeneous, trade could not occur at a single ratio of buyers to sellers if the discount rate is positive and instead an equilibrium price function would arise (see discussions by Mortensen and Wright, 1997, Mortensen and Pissarides, 1999, pp. 2589-2591, and Sattinger, 2000). Mortensen and Wright (1997) show that the dispersion in prices and ratio declines to zero as the discount rate approaches zero.

Section II develops the model, including matching function and behavior of buyers, sellers and brokers. Section III determines the conditions for long run equilibrium. Section IV then develops the differential equations determining the price dynamics. Section V presents conclusions and discusses extensions.

2. Model

2.1. Matching Technology

Suppose that buyers and sellers in a market must find each other before a trade can take place. Let $M(x, y)$ be the rate at which trades (or matches) occur between buyers and sellers if there are x buyers and y sellers. When buyers and sellers find trading partners without a broker, they are price takers and the trade takes place at the current price P. Assume $M(x,y)$ is homogeneous of degree one and has continuous first and second order partial derivatives. Assume further that

$$M_1(x,y) = \partial M(x,y)/\partial x > 0 \text{ if } y > 0 \tag{2.1}$$

$$M_2(x,y) = \partial M(x,y)/\partial y > 0 \text{ if } x > 0 \tag{2.2}$$

$$M_{11}(x,y) = \partial^2 M(x,y)/\partial x^2 < 0 \tag{2.3}$$

$$M_{22}(x,y) = \partial^2 M(x,y)/\partial y^2 < 0 \tag{2.4}$$

$$M(0,y) = M(x,0) = 0 \tag{2.5}$$

The conditions on $M_{11}(x,y)$ and $M_{22}(x,y)$ reflect diminishing marginal products of buyers and sellers in the production of trades, which are standard assumptions in neoclassical production theory. The other conditions are not required in neoclassical production theory but are reasonable in the context of the production of trades. The conditions that $M_1(x,y) > 0$ if $y > 0$ and $M_2(x,y) > 0$ if $x > 0$ specify that adding buyers or sellers always raises the number of trades if there are already trading partners in the market. Also, the condition $M(0,y) = M(x,0) = 0$ is imposed because no trades can take place if there are no trading partners of one type.

Since $M(x,y)$ is homogeneous of degree one,

$$M(x,y) = yM(x/y,1) \tag{2.6}$$

In the absence of a broker, if there are Q_D buyers and Q_S sellers, the number of trades per unit of time will be $M(Q_D,Q_S) = Q_S M(Q_D/Q_S,1) = Q_S M(\theta,1)$, where $\theta = Q_D/Q_S$. The rate at which a buyer gets a good is $M(\theta,1)Q_S/Q_D = M(\theta,1)/\theta$, and the rate at which a supplier sells the good is $M(\theta,1)Q_S/Q_S = M(\theta,1)$. To simplify notation, let $m(\theta) = M(\theta,1)$.

2.2. Buyers

Suppose a buyer holds at most one unit of a durable good at a time and obtains instantaneous utility $V_G e^{-\delta a}$ from a good that is now a units of time old. Suppose V_G is the same for all buyers. If the buyer plans on replacing the good at age μ, the asset value of holding a good of age a is

$$\begin{aligned} W_{BG}(a,P,F,\theta,\mu) &= \int_a^{\mu} e^{-r(x-a)}V_G e^{-\delta x}dx + e^{-r(\mu-a)}W_B(P,F,\theta,\mu) \\ &= \frac{e^{-\delta a}-e^{-\delta\mu-r(\mu-a)}}{r+\delta}V_G + e^{-r(\mu-a)}W_B(P,F,\theta,\mu) \quad (2.7) \end{aligned}$$

where r is the discount rate (the same for all agents) and $W_B(P,F,\theta,\mu)$ is the asset value of being in the market for the good when the price is P, broker fee is F (if a broker is used; otherwise $F=0$) and the ratio of buyers to sellers is θ. The flow of asset value from being in the market is

$$rW_B(P,F,\theta,\mu) = -c_b + \frac{m(\theta)}{\theta}\left(W_{BG}(0,P,F,\theta,\mu) - W_B(P,F,\theta,\mu) - P - F\right) \quad (2.8)$$

where c_b is the cost per period of being in the market, $m(\theta)/\theta$ is the rate of finding a seller per unit of time, P and F are the price to the seller and fee to the broker paid by the buyer if a seller is found, and $W_{BG}(0,P,F,\theta,\mu)$ is the asset value of having a new good. The fee can be positive or negative, or zero if no broker is used. Substituting $W_{BG}(0,P,F,\theta,\mu)$ from 2.7 into 2.8 and solving for $W_B(P,F,\theta,\mu)$ yields

$$W_B(P,F,\theta,\mu) = \frac{\left((V_G(1-e^{-\mu(r+\delta)})/(r+\delta)) - (P+F)\right)(m(\theta)/\theta) - c_b}{r+(1-e^{-r\mu})m(\theta)/\theta} \quad (2.9)$$

This is the buyer's objective function when in the market. The construction of the asset equations for the buyer assume that the buyer at each point in time has static expectations that the variables P, F, θ and μ will have the same values in the future as they have now. An important extension is to develop asset equations that incorporate expectations of changes in prices and other variables. Optimizing $W_B(P,F,\theta,\mu)$ with respect to μ (or equivalently optimizing $W_{BG}(0,P,F,\theta,\mu)$ with respect to μ) yields a first order condition for the optimal time to reenter the market. The optimal age of the good, μ, satisfies

$$W_B(P,F,\theta,\mu) = e^{-\delta\mu}V_G/r \quad (2.10)$$

When this condition holds, the flow of asset value from being in the market to buy a new good equals the flow of utility from the old good. The second order condition can be shown to hold.[2] The calculation of optimal μ is only relevant to the decision to buy a new good when the buyer is on the verge of reentering the market. However, calculations of μ are also relevant to the value of a new good when the buyer is in the market. Because of the exponential function involving μ, the optimal age of the good cannot be found analytically.[3] However, it is possible to solve for the price at which a given value of μ is optimal. Let $P_\mu(\theta, F, \mu)$ be the price that satisfies the first order condition for μ, given θ, F, and μ.

By setting $W_B(P, F, \theta, \mu)$ equal to a constant and solving for F, it is possible to obtain a particular indifference curve for the buyer. Let $MRS_B(P, F, \theta, \mu)$ be the buyer's marginal rate of substitution of fee for ratio θ; this is the same as the absolute value of the slope of the buyer's indifference curve at a given point (taking P and μ as given). For a general matching function satisfying 2.1 through 2.5, it is ambiguous whether the indifference curve (between F and θ) is concave. However, the indifference curve is unambiguously concave for a constant elasticity of substitution matching function with positive elasticity of substitution.

Let Q_{Dt} be the number of buyers in the market at time t and let Q_B be the total number of buyers, with and without the good. The effect of changes in μ on numbers reentering is shown in Figure 2.1. If μ is constant, the line at $t-\mu$ moves right at the same rate as time passes, so that the number of buyers reentering is just the number of buyers in the market μ time units previously, $Q_{Dt-\mu}$, times the rate at which they got trades, $m(\theta_{t-\mu})/\theta_{t-\mu}$, where $\theta_{t-\mu}$ is the ratio of buyers to sellers at time $t-\mu$. If μ is increasing, the line at $t-\mu$ moves right at a slower speed than the line at t, and the rate at which buyers reenter is reduced to

$$Q_{Dt-\mu}(m(\theta_{t-\mu})/\theta_{t-\mu})\frac{d(t-\mu)}{dt} = Q_{Dt-\mu}(m(\theta_{t-\mu})/\theta_{t-\mu})\left(1-\frac{d\mu}{dt}\right) \tag{2.11}$$

If μ is decreasing, the line moves rightward faster than t, and the number reentering is greater than $Q_{Dt-\mu}(m(\theta_{t-\mu})/\theta_{t-\mu})$. The rate of change of Q_{Dt} in general is

$$\frac{dQ_{Dt}}{dt} = Q_{Dt-\mu}(m(\theta_{t-\mu})/\theta_{t-\mu})\frac{d(t-\mu)}{dt} - Q_{Dt}(m(\theta_t)/\theta_t) + Q_{Nt} \tag{2.12}$$

[2] Suppose the buyer considers extending the length of time that the current good will be held, holding future optimal ages constant. Then the value of being in the market, $W_B(P, F, \theta, \mu)$, will be unaffected by the extension. The second derivative of $W_{BG}(a, P, F, \theta, \mu)$ with respect to μ, evaluated at $a = \mu$, is $-\delta e^{-\delta\mu}V_G$, satisfying the second order condition for a maximization.

[3] Series expansions of the exponential functions around their expected values can be used to obtain accurate analytical expressions for the optimal age.

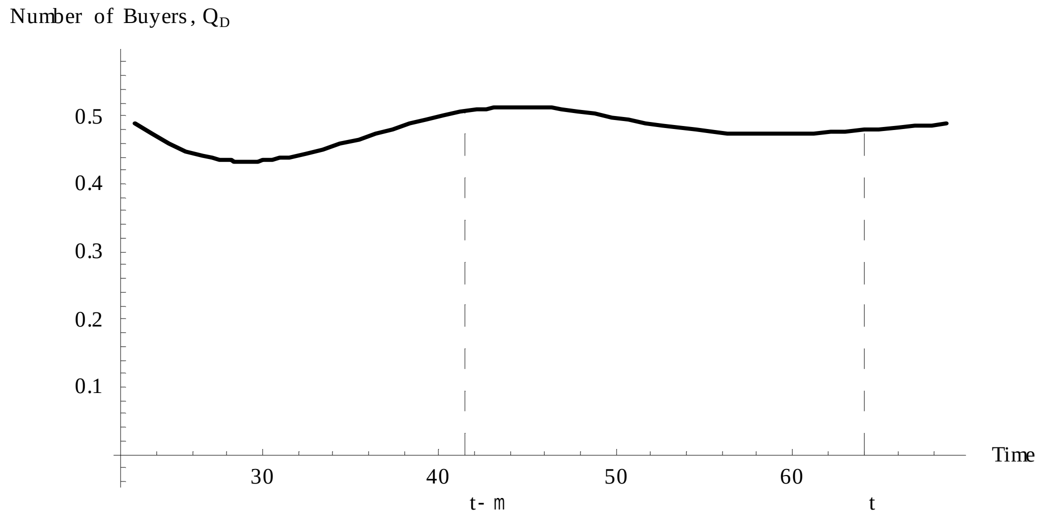

Figure 2.1: Number of Buyers in Market

where Q_{Nt} is the rate of entry or exit of buyers from the economy and $Q_{Dt}(m(\theta_t)/\theta_t)$ is the rate at which current buyers get the good and leave the market. In the dynamic analysis to be undertaken in Section 4, Q_{Nt} provides a means by which a disequilibrium, equivalent to a shift in demand, can be analyzed.

2.3. Sellers

Sellers produce one good at a time at cost V_S, the same for all sellers. After a sale, sellers immediately reenter the market. A seller's expected profit at a point in time is

$$W_S(P, F, \theta) = -c_s + m(\theta)(P + F - V_S) \tag{2.13}$$

where c_s is the cost of participating in the market, P is the price received for the good, F is the fee paid by a broker for the seller's participation, $m(\theta)$ is the rate of getting trades per unit of time, and θ is again the ratio of buyers to sellers. The fee F could be zero, if no broker is used, or it could be positive or negative.

Setting $W_S(P, F, \theta)$ equal to a constant and solving for F yields the seller's indifference curve between F and θ. Let $MRS_S(P, F, \theta)$ be the seller's marginal rate of substitution of F for θ. For a general matching function, the indifference curve for sellers is unambiguously convex.

Let Q_S be the number of sellers on the market. Suppose the number of sellers is an increasing function of $W_S(P, F, \theta)$:

$$Q_S = S_0 \left(W_S(P, F, \theta)\right)^{\beta}, S_0, \beta > 0 \tag{2.14}$$

(Alternatively, the number of sellers could be taken to be constant.)

2.4. Requirements for Trade

For trade to occur, parameter values must be such that $W_B(P, F, \theta, \mu) > 0$ and $W_S(P, F, \theta) > 0$. From 2.8 (after solving for $W_B(P, F, \theta, \mu)$) and 2.13, it follows that

$$W_{BG}(0, P, F, \theta, \mu) - P - F \geq \frac{c_b \theta}{m(\theta)} \tag{2.15}$$

and

$$P + F - V_S \geq \frac{c_s}{m(\theta)} \tag{2.16}$$

From 2.10, as $W_B(P, F, \theta, \mu)$ approaches zero, μ increases indefinitely. Buyers hold on to the old good since buying a new good yields a negligible increase in asset value. A lower bound for $W_{BG}(0, P, F, \theta, \mu)$ arises when $W_B(P, F, \theta, \mu) = 0$; then $W_{BG}(0, P, F, \theta, \mu) = V_G/(r + \delta)$, the value of getting a new good and holding it indefinitely. Substituting this lower bound into 2.15 and combining the inequalities yields

$$\frac{V_G}{r + \delta} - V_S \geq \frac{c_b \theta + c_s}{m(\theta)} \tag{2.17}$$

The term on the right in this condition is the cost in participation fees per trade. Its minimum value depends on $m(\theta)$ and occurs when

$$\frac{m'(\theta)}{m(\theta) - \theta m'(\theta)} = \frac{\partial M / \partial Q_D}{\partial M / \partial Q_S} = \frac{c_b}{c_s} \tag{2.18}$$

When this condition holds, the contributions of marginal buyers and sellers to trades are proportional to participation costs. The condition in 2.17 imposes restrictions on $V_G, V_S, \delta, r, c_b, c_s$ and the matching function that must be satisfied for trade to occur. As demonstrated by Mortensen and Wright (1997), there is a sufficiently high interest rate at which no trades will occur.

2.5. Brokers

Brokers operate by offering alternative ratios of buyers to sellers and charging fees, using the same matching technology available to buyers and sellers in the absence of brokers. Assume that brokers incur no added costs in operating. To see how brokers can bring about equilibrium in the market for access to trading partners, suppose buyer and seller indifference curves (between the fee and the ratio of buyers to sellers) intersect at the prevailing price. Then a broker can make a profit by offering a ratio at which the fee a buyer is willing to pay exceeds the fee a seller is willing to receive in order to engage in trade at that ratio. (Because the indifference curves intersect, this must occur at either a higher ratio or a lower ratio.) Free entry of profit-maximizing brokers implies that the indifference curve for buyers is nowhere above the indifference curve for sellers. The fee charged to buyers will then equal the fee paid to sellers (or else the fee charged to sellers will equal the fee paid to buyers). This occurs when the indifference curve for buyers is tangent to the indifference curve for sellers, at which point profits are eliminated. At the point of tangency, buyer and seller trade-offs between fee and ratio of buyers to sellers are equal and any search externalities of buyer or seller entry are eliminated.[4]

Although the distinction between price and fee is useful in describing the behavior of brokers, it is more convenient to work with the sum in calculating price dynamics. This is possible since P and F enter the buyer and seller asset equations in the same way, so that $W_B(P, F, \theta, \mu) = W_B(P + F, 0, \theta, \mu)$ and $W_S(P, F, \theta) = W_S(P + F, 0, \theta)$. Let $p = P + F$. The condition that the sum of price and fee is such that the indifference curves are tangent can be found analytically by setting $MRS_B(p, 0, \theta, \mu)$ equal to $MRS_S(p, 0, \theta)$ and solving for p. Let $p_{\tan}(\theta, \mu)$ be the value of p at which the indifference curves are tangent, given θ and μ. Also, let $W_S(p, \theta) = W_S(p, 0, \theta)$ and $p_\mu(\theta, \mu) = P_\mu(\theta, 0, \mu)$.

3. Long Run Equilibrium

Long run equilibrium can be determined when the rate of entry or exit of buyers from the economy, Q_{Nt}, is zero. The conditions for long run equilibrium are:

1. Buyers optimally choose the age of the good at reentry.

[4] Mortensen and Wright (1997) show that tangent indifference curves satisfy the Hosios condition for absence of search externalities (Hosios, 1990).

2. Competitive brokers eliminate all profitable brokerage opportunities, so that buyers and sellers have the same marginal rates of substitution between the price and the ratio of buyers to sellers.

3. The ratio of buyers to sellers, θ, equals Q_D/Q_S, where Q_D and Q_S are determined endogenously.

4. The rate of change of Q_D is zero for at least the last μ units of time.

The values of the variables in long run equilibrium can be determined as follows. Conditions 1 and 2 above hold when

$$p = p_{\mu}(\theta, \mu) \tag{3.1}$$

and

$$p = p_{\tan}(\theta, \mu) \tag{3.2}$$

respectively. Let Q_B be the total number of buyers, either in the market or with the good. Then in the long run, taking the sum,

$$Q_B = Q_D + \mu Q_D m(\theta)/\theta \tag{3.3}$$

or

$$Q_D = \frac{Q_B}{1 + \mu m(\theta)/\theta} \tag{3.4}$$

From condition 4, using 2.13 and 3.4,

$$\theta = \frac{Q_B/(1 + \mu m(\theta)/\theta)}{S_0 \left(W_S(p, \theta)\right)^{\beta}} \tag{3.5}$$

Then 3.1, 3.2 and 3.5 can in general be solved for p, θ and μ. Existence and uniqueness arise if the relation between p and θ generated by the tangency condition (with μ adjusting optimally) intersects the relation generated by 3.5, again with μ adjusting optimally, at one and only one point.[5]

In the example worked out in the following section on price dynamics, the long run equilibrium is given by $p = 8.45$, $\theta = 1$, $\mu = 20$ and $Q_B = 3.85$.[6]

[5] Existence and uniqueness are proven formally in Sattinger (1999) using a simpler model of buyer behavior.

[6] The other parameters are $r = .1$, $\delta = .15$, $V_S = 8$, $c_b = .1$, $c_s = .05$, $V_G = 2.51$, $Q_B = 3.85$, $s0 = 1$ and $\beta = 1$. The matching function is given by $m(\theta) = \theta/(1 + \theta)$. This is a constant elasticity of substitution production function with elasticity equal to one-half. In practice, since an analytic solution for μ cannot be given, μ and θ are chosen and the system is solved for p, V_G and Q_B.

4. Short Run Dynamics

This section develops the differential equations that hold for p, θ, μ and Q_D at each point in time. In the short run, conditions 1, 2 and 3 in Section 2.6 are assumed to hold, but condition 4 does not necessarily hold. Then changes in Q_D over time generate fluctuations in p, θ and μ. Let

$$\begin{aligned} EQ1 &= p - p_{\mu}(\theta, \mu) && (4.1) \\ EQ2 &= p - p_{\tan}(\theta, \mu) && (4.2) \\ EQ3 &= \theta - Q_D / \left(S_0 \left(W_S(p, \theta) \right)^{\beta} \right) && (4.3) \end{aligned}$$

where the arguments of $EQ1$, $EQ2$ and $EQ3$ (which are p, θ, μ and Q_D) are suppressed to simplify notation. Since $EQ1$, $EQ2$ and $EQ3$ are zero at each point in time, they yield three conditions on the derivatives dp/dt, $d\theta/dt$, $d\mu/dt$ and dQ_D/dt. A fourth condition arises from 2.12. Let A be the four by four matrix

$$A \equiv \begin{pmatrix} \partial EQ1/\partial p & \partial EQ1/\partial \theta & \partial EQ1/\partial \mu & \partial EQ1/\partial Q_{Dt} \\ \partial EQ2/\partial p & \partial EQ2/\partial \theta & \partial EQ2/\partial \mu & \partial EQ2/\partial Q_{Dt} \\ \partial EQ3/\partial p & \partial EQ3/\partial \theta & \partial EQ3/\partial \mu & \partial EQ3/\partial Q_{Dt} \\ 0 & 0 & (m(\theta_{t-\mu})/\theta_{t-\mu}) Q_{Dt-\mu} & 1 \end{pmatrix}$$

Let R be the four by one vector

$$R \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ (m(\theta_{t-\mu})/\theta_{t-\mu}) Q_{Dt-\mu} - (m(\theta_t)/\theta_t) Q_{Dt} + Q_{Nt} \end{pmatrix}$$

Let D be the four by one vector

$$D \equiv \begin{pmatrix} dp/dt \\ d\theta/dt \\ d\mu/dt \\ dQ_D/dt \end{pmatrix}$$

Then $AD = R$ and $D = A^{-1}R$. Since only the fourth entry of R is nonzero, the derivatives only depend on the fourth column of A^{-1}. The derivatives will all have the same sign and will be proportional to the fourth entry of R at each point in time. Whenever $dQ_D/dt = 0$, the other derivatives will also be zero, even if the market is not in long run equilibrium. The derivatives $A^{-1}R$ can be derived analytically so the derivatives can be found at each point in time.

Figures 4.1 through 4.4 present two dynamic solutions resulting from an increase and a decrease in demand. Figure 4.1 shows the price dynamics, with both

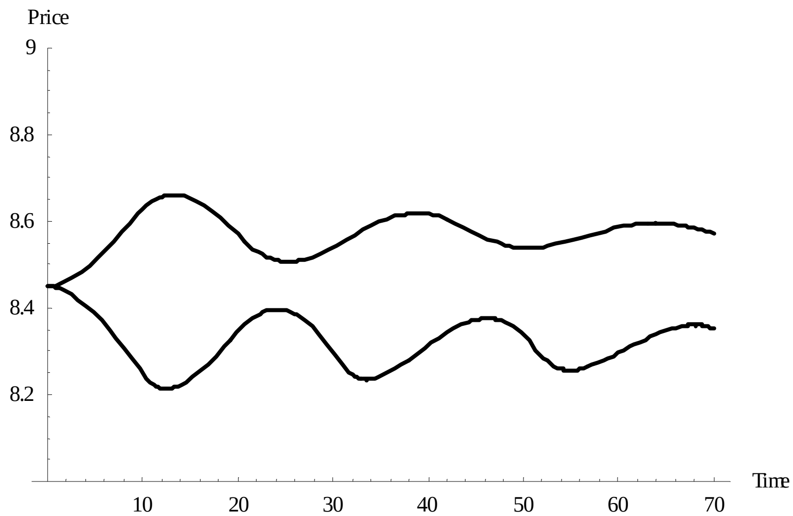

Figure 4.1: Price Dynamics from Shifts in Demand

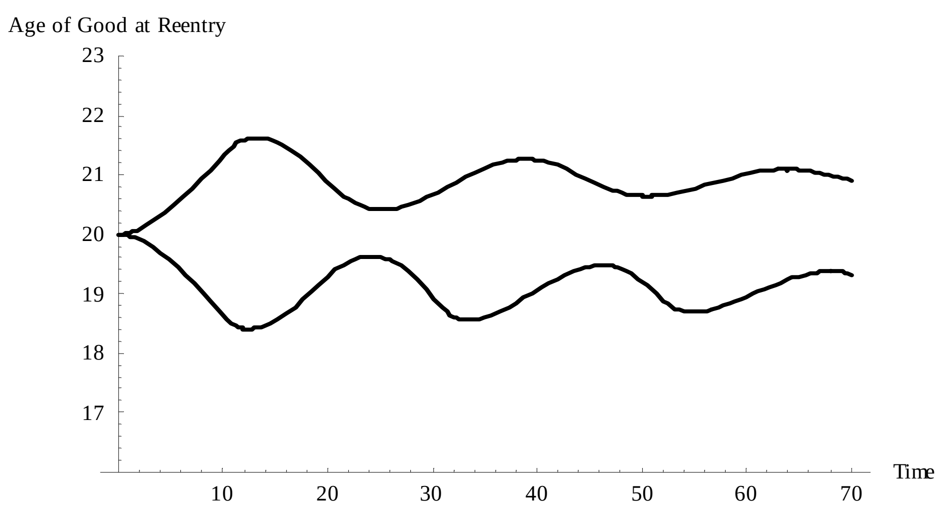

Figure 4.2: Age of Good at Reentry

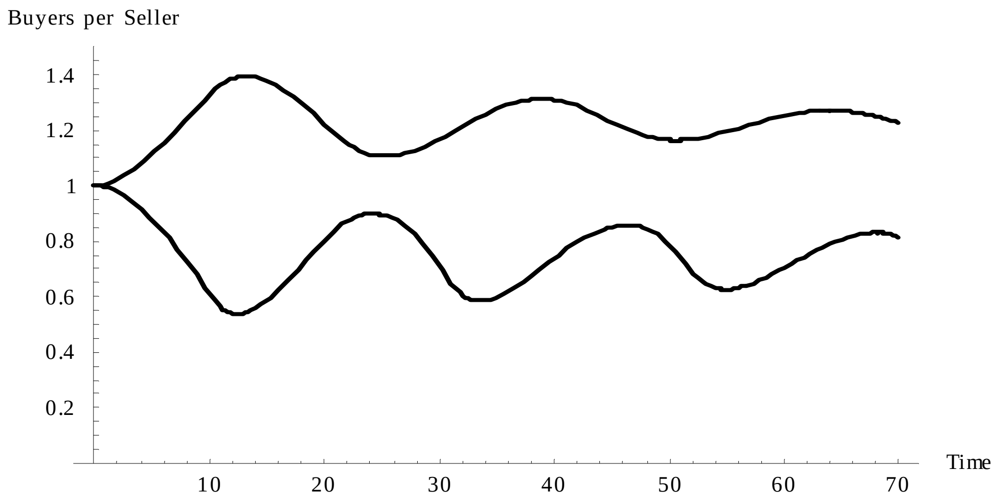

Figure 4.3: Ratio of Buyers to Sellers

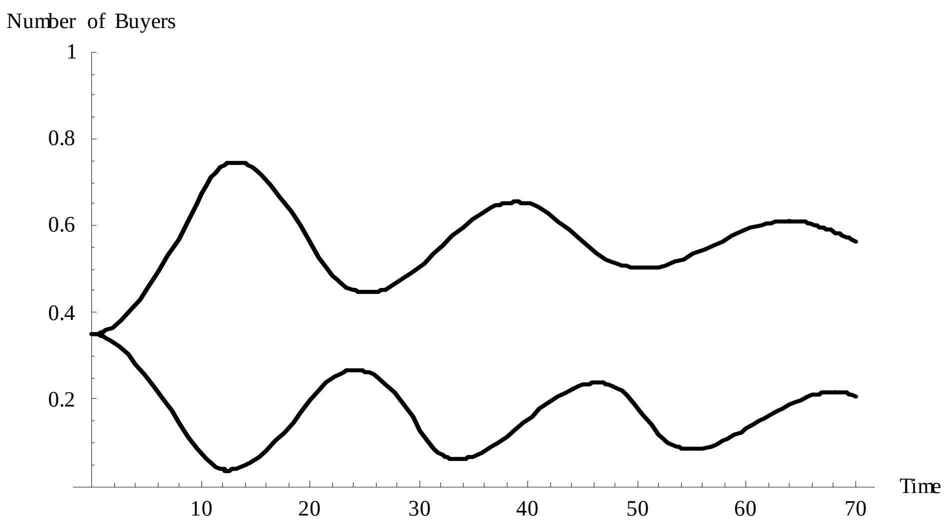

Figure 4.4: Number of Buyers in Market

paths starting from the same long run equilibrium. Figure 4.2 shows the optimal age of the good, Figure 4.3 shows the ratio of buyers to sellers, θ, and Figure 4.4 shows the number of buyers on the market, Q_D. The upper curves show the response to an increase in demand, and the lower curves show the response to a decrease in demand. The solutions assume the system is in long run equilibrium (described in the previous section) for the previous 20 units of time. Then at time 0, for the demand increase, Q_{Nt} increases linearly from 0 to 0.2 at time 10, and then declines linearly from .2 to 0 at time 20, after which it is zero. For the demand decrease, Q_{Nt} decreases linearly from 0 to -.2 at time 10, and then increases from -.2 to 0 at time 20, after which it is zero. These demand shifts change Q_B by .2.

Because $Q_{Dt-\mu}$ and $\theta_{t-\mu}$ stay the same until $t - \mu = 0$, there is initially no feedback from the new entrants to the number reentering. For the demand increase, as Q_D increases from the new entrants, p and θ go up. Buyers holding goods respond by raising μ, so that fewer reenter the market even though $Q_{Dt-\mu}$ is constant for some time. The reduction in reentrants moderates the increase in current number of buyers, Q_{Dt}, from the new entrants. As θ increases, the rate of matching for buyers, $m(\theta)/\theta$, declines until $dQ_D/dt = 0$. Then since Q_{Nt} is declining in this example, dQ_D/dt goes negative, leading to reductions in p, θ and μ. At $t = 20$, the number of buyers continues to decline because no more new entrants are coming in, although the number reentering is increasing because μ is declining. At $t = 20.75$, the optimal age of the good is also 20.75. Then the buyers who got the good at $t = 0$, when disequilibrium started, start to reenter the market. Their effect is to raise the current number of buyers in the market. After $t = 20$, what happens at a peak or trough depends on what is happening to the number reentering, $(m(\theta_{t-\mu})/\theta_{t-\mu})Q_{Dt-\mu}$. Eventually, the market approaches a new equilibrium, with higher price, ratio of buyers to sellers, age of good at reentry, and number of buyers in the market. Although the distribution of buyers by age of good is initially uneven after entry or exit of buyers, the effect of the price fluctuations appears to be to smooth out that distribution. The effects of a demand decrease of the same magnitude are not symmetric to the effects of a demand increase. Because μ is reduced by a demand decrease, buyers from $t = 20$ reenter at an earlier point in time, so that the troughs of the variables from a demand decrease occur before the peaks from the demand increase.

The purpose of these derivations is to demonstrate that at each point in time, the price is determined by the market for access to trading partners, implemented by broker profit-seeking activity. Application of the equilibrium conditions for the market for access to trading partners provides a model of disequilibrium price determination (disequilibrium in the sense that the long run condition $dQ_{Dt}/dt = 0$ is not satisfied).

5. Conclusions

In the model developed in this paper, the price at any point in time is determined by the equilibrium in the market for access to trading partners instead of by the condition that quantity demanded equal quantity supplied. The market for access to trading partners operates even when the market for the good is in disequilibrium, in the sense that the number of buyers is changing over time. It is generated by the actions of profit-seeking competitive brokers, whose fees set implicit prices for the contacts between trading partners.

A market for access to trading partners implemented by brokers solves several problems in the determination of price in a single market. First, it explains how the price changes in a market when buyers and sellers are all price takers. Brokers are motivated by profit-maximization to set fees that effectively change prices whenever the market is in disequilibrium. Compared to brokers, auctioneers in a Walrasian system have no profit motive for their function. While brokers have the power to set fees, they are competitive in the sense of being buyer and seller status takers, i.e., they cannot affect the asset levels of buyers and sellers in the market.

Second, the price mechanism is the same in equilibrium as in disequilibrium. It is therefore unnecessary to develop a separate model to explain price determination in disequilibrium, when buyers and sellers would have some degree of monopoly and monopsony power in the absence of broker activity. In the broker model, buyers and sellers continue to exhibit competitive price-taking behavior, even when the market for the good is in disequilibrium.

Finally, the market for access to trading partners implemented by brokers generates dynamic paths of adjustment of price. Since the fee generated by brokers determines the change in price, the differential equation relating rate of change in price to the price itself can be found, along with the other differential equations in the system. In contrast, the Walrasian model requires additional assumptions to generate dynamics (e.g., the rate of change of prices is proportional to excess demand or supply).

The model emphasizes the important role of brokers in generating a market for an economic good (access to trading partners) that would otherwise not exist. In the absence of broker activities, buyers and sellers would have no simple mechanism for trading money for access to trading partners.

The models developed here are based on naive, static expectations. Buyers make decisions about reentry based on expectations that prices and conditions of trade will continue indefinitely. Such expectations are counterfactual in a model of price dynamics. The next step is to incorporate price and market condition changes into optimal buyer behavior in the generation of price dynamics.

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