Price Dispersion and Short Run Equilibrium in a Queuing Model*

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May 7, 2003

Abstract

Price dispersion is analyzed in the context of a queuing market where customers enter queues to acquire a good or service and may experience delays. With menu costs, price dispersion arises and can persist in the medium and long run. The queuing market rations goods in the same way whether firm prices are optimal or not. Price dispersion reduces the rate at which customers get the good and reduces customer welfare.

*The author is indebted to Guillaume Rocheteau, Robert Shimer and participants at the Search session of the Society for Economic Dynamics meetings June 28-30, 2002.

1. Introduction

This paper analyzes price dispersion in a queuing market where customers may experience delays in waiting to be served. The price dispersion arises and persists because queue lengths compensate for price differences, and firms limit price changes in response to menu costs. The paper considers consequences of price dispersion for arrival rates, efficiency and consumer welfare. Queue rationing occurs through customer choice of queues rather than price adjustment. It operates in the same manner whether the economy's equilibrium is short run (with arbitrary and possibly suboptimal firm prices), medium run (with optimal firm prices but with firm profits and losses) or long run (with optimal prices and zero profits).

The queuing market considered here is an extension of Sattinger (2002), which analyzed medium and long run equilibria in the absence of price dispersion.[1] This paper differs from the earlier paper by introducing menu costs, which explain how price dispersion can arise and persist.[2] It also extends the description of queue rationing with price dispersion to the short run.

The major conclusions are as follows. With menu costs, price dispersion can arise in short, medium and long run equilibria. Price dispersion causes inefficiency in a queuing market by reducing the aggregate rate at which customers in the market get the good or service. The queuing mechanism allocating goods or services to customers operates whether or not firm prices are in equilibrium (i.e., are optimal for firms). Price dispersion and disequilibrium then have minimal consequences for the operation of a queuing market, in contrast to standard market models.

The basic results from a single price queuing market equilibrium (without menu costs) are as follows. Consider a market in which customers acquire a good or service by waiting in a queue at a firm. Let α be the arrival rate of customers at firms, and let σ be the rate at which customers in a queue are serviced or provided with the good. Then $\rho = \alpha/\sigma$ is the erlang or traffic intensity, and $1 - \rho$ is the expected proportion of time a firm is idle (with no customers in line). The expected number of customers in a firm's queue is $\rho/(1 - \rho)$, and the average rate at which customers in the market get the good or service is $\sigma - \alpha$.[3] Firms

[1] Queuing theory is treated in Cox and Smith, 1961; Feller, 1957; Karlin, 1966; Karlin and Taylor, 1981; Saaty, 1961; and Taylor and Karlin, 1984.

[2] See Andersen, 1994, for a review of the menu cost literature.

[3] Sattinger (2002, p. 536) refers to $\sigma - \alpha$ as the rate at which a customer already in a queue gets the good. Guillaume Rocheteau points out that customers already in a queue will experience different rates.

and customers have a common interest rate r. The good or service deteriorates at the rate μ. A customer chooses a queue on the basis of the expected value of being in the queue, which equalizes the expected value across firm queues even if price differences are present. Let V_{NG} be the expected value of being in a queue. Solving the asset value equations for a customer moving between having the good and not having the good yields

$$V_{NG} = \frac{(\sigma - \alpha)(\phi - (r + \mu)P) - \omega(r + \mu)}{r(r + \mu + \sigma - \alpha)} \tag{1.1}$$

where ϕ is the value of the good to customers and ω is the cost to customers of being in the market. The steady state arrival rate α is such that flows of customers getting the good equal flows of customers entering the market to get the good, and is given by

$$\alpha = \left(\sigma + \mu + \mu \frac{N_C}{N_F} - \sqrt{(\sigma + \mu + \mu \frac{N_C}{N_F})^2 - 4\mu\sigma \frac{N_C}{N_F}} \right) /2 \tag{1.2}$$

where N_C is the measure of customers and N_F is the measure of firms. Firms optimally choose a price to maximize expected profits subject to the response of the arrival rate at the firm that yields the same value of V_{NG} as at other firms. If a firm charges price $P_{i,}$ its arrival rate would be

$$\alpha_i = \alpha(P_i, V_{NG}) = \sigma - \frac{(r + \mu)(rV_{NG} + \omega)}{\phi - (r + \mu)P_i - rV_{NG}} \tag{1.3}$$

The expected profit rate of a firm charging price P_i is

$$\pi = \alpha_i(P_i - C) - k \tag{1.4}$$

where C is the cost of producing the good and k is the cost per period of being in the market. The optimal price is given by

$$P = C + \frac{\alpha(\phi + \omega - (r + \mu)C)}{(r + \mu)\sigma + (\sigma - \alpha)^2} \tag{1.5}$$

This price equalizes the slopes of the customer indifference and firm isoprofit curves between arrival rate and price.

Various forms of disequilibrium can arise involving adjustments to stochastic changes, incomplete customer information about firm expected queue lengths or pricing, and unequal customer values of entering firm queues. For the purposes of this paper, a short run equilibrium is defined as occurring when the expected value of entering a firm's queue is the same for all firms. However, in the short run, firms may not have adjusted their prices optimally to current market conditions. The short run therefore describes a market with optimal customer responses to potentially suboptimal firm pricing. The medium run occurs when firms have adjusted prices to current steady-state conditions but have not made entry and exit decisions in response to profits or losses. The long run occurs when, in addition to medium run conditions, firms are making zero profits and seek neither to enter nor to exit.

The next section describes how menu costs can generate price dispersion in the queuing market. Section 3 shows how dispersion in prices leads to dispersion in arrival rates so that the expected value of queuing at different firms is the same. The section shows the effects of price dispersion on the aggregate arrival rate, efficiency and customer welfare. Section 4 draws conclusions about the role of price dispersion in queuing markets.

2. Menu Costs and Price Dispersion

Equilibrium price dispersion can arise in the presence of menu costs. Let M be the menu costs of changing the price. A price change must then raise the rate of firm profits by at least rM for a price change to be profitable. The upper and lower prices at which a price change is just profitable can be found from the condition 1.3 determining the arrival rate at a firm charging arbitrary price P_i, in combination with the firm's profit function in 1.4. Substituting α_i from 1.3 into 1.4 yields the firm's profits as a function of the price charged. This is shown in Figure 2.1 using specific parametric values.[4] Subtracting rM from the maximum profit level yields the horizontal line in Figure 2.1. At the intersections of the horizontal line with the profit curve, the profit gain just equals the menu costs per period. At prices between the vertical dashed lines, the firm would lose money by changing the price and would therefore leave the price unchanged. At prices outside the dashed lines, the firms could gain by changing the price.

The interval within which prices remain unchanged can be determined as a function of the menu costs. Setting firm profits (with α_i determined by 1.3) equal to profits at the optimal price minus menu costs yields two solutions for the price, corresponding to the upper and lower limits of the interval. Varying the menu

[4] The figure assumes $\sigma = 2$, $r = .1$, $\mu = .2$, $\phi = 6$, $N_C = 100$, $N_F = 10$, $C = 10$, $M = 1$, $\omega = 1$, and $k = 2$.

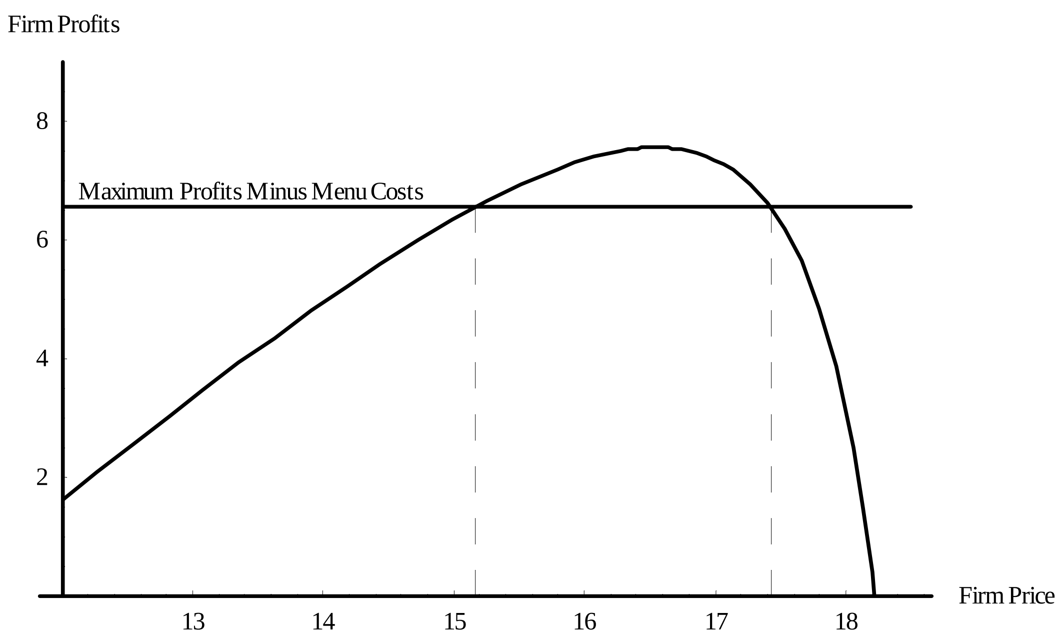

Figure 2.1: Firm Profits Versus Price

costs generates different upper and lower limits within which prices would not be changed. As menu costs increase, the interval grows, but not in proportion to the menu costs. At menu costs close to zero, the interval grows rapidly, reflecting the zero slope of the profit function at the maximum profit in Figure 2.1. Negligible menu costs can therefore permit a substantial amount of price dispersion. The lower limit falls more rapidly than the upper limit rises as menu costs increase, because high prices have greater profit consequences than low prices.

The particular distribution of prices within an interval is not determined by this static analysis. The optimal dynamic price setting strategy of a firm in a stochastic environment, with arrival rates varying from period to period, would not generally result in price changes exactly at the upper and lower limits shown in Figure 2.1. However, the analysis establishes that price dispersion would not be eliminated by firm profit maximization in the presence of menu costs. The dispersion could be relatively large even for small menu costs.

In a dynamic analysis of a queuing market, price changes would occur when firms decide that profit increases would be sufficient to cover menu costs. Suppose firms establish threshold positive and negative differences from the optimal price at which they change prices.[5] If the optimal price is declining, only firms at the upper threshold would lower their prices. If the price is increasing, only firms at the lower threshold would raise their prices. The density of firms at a given price could be generated iteratively from the probabilities of threshold prices leading firms to change to the given price. It is unlikely that average price changes would be proportional to stochastic changes in numbers of customers, or that prices would be uniformly distributed within a range.

3. Dispersion in Prices and in Arrival Rates

This section describes how the queuing market would operate with dispersion in prices and arrival rates. The section then shows how equilibrium can be determined when there is price dispersion. Dispersion affects the equilibrium by raising the aggregate number of customers in line for a given average arrival rate. It reduces the proportion of customers with the good and makes customers worse off. Aggregate firm profits are reduced, mostly because of suboptimal prices; but a

[5] It is possible that depending on the stochastic behavior of the number of customers, firms would not change prices within a range wider than when menu costs are exactly recovered. Also, in a stochastic environment, firms would not necessarily change prices to the current optimal level.

firm charging the optimal price would make greater profits. The optimizing problems facing firms and customers are substantially the same whether or not there is dispersion.

Price dispersion affects the queuing market through unequal arrival rates. If there is a system with multiple queues that have the same service times (at supermarket check-out lines, for example), the expected aggregate number in the queues will be minimized when the arrival rates at each queue are the same. Unequal arrival rates raise the expected number waiting. This point can be demonstrated as follows. As before, let α_i be the arrival rate at firm i and suppose the completion rate σ is the same at all firms. Then the erlang or traffic intensity at firm i is $\rho_i = \alpha_i/\sigma$. The expected number of customers in line for firm i is then $\rho_i/(1-\rho_i) = \alpha_i/(\sigma-\alpha_i)$. If the α_i are unequal and the average value is α, then[6]

$$\sum_{i=1}^{N_F} \frac{\alpha_i}{\sigma - \alpha_i} > \frac{\alpha}{\sigma - \alpha} N_F \tag{3.1}$$

The reason for the inequality is the appearance of α_i in the denominator, so that when α_i is above average, $\sigma - \alpha_i$ is low and the ratio is boosted in value.[7]

In standard market models, allocation of goods from sellers to buyers is tied to the price determination mechanism.[8] In a queuing market, firm idle times reconcile market service capacity with customer arrival rates. This rationing mechanism operates separately from the price determination process. A queue rationing allocation can then be determined for an arbitrary distribution of prices. Define a queue rationing allocation as occurring when queue lengths at firms equalize the value of entering each firm's queue and the aggregate arrival rate equalizes flows of customers into and out of the market. A queue rationing allocation continues indefinitely until firms change their prices, or until some other change in the

[6]Since the number in line at one firm is not independent of the number in line at another firm, summing the expected numbers in line does not necessarily yield the expected aggregate number in line. However, it will be approximately the same and will be taken to equal the expected aggregate number for purposes of the analysis here.

[7]This inequality can first be shown for two firms. Suppose $\alpha = (\alpha_1 + \alpha_2)/2$, with α_1 and α_2 each less than σ. Then

$$\frac{\alpha_1}{\sigma - \alpha_1} + \frac{\alpha_2}{\sigma - \alpha_2} = \frac{2\alpha(\sigma - \alpha) + (\alpha_1 - \alpha_2)^2/2}{(\sigma - \alpha)^2 - ((\alpha_1 - \alpha_2)/2)^2}$$

This exceeds $2\alpha/(\sigma - \alpha)$ whenever $\alpha_1 \neq \alpha_2$. The inequality can then be established for any number of firms by sequential averaging.

[8]See Carlton (1989) for a review of market clearing.

market occurs (e.g., the number of customers increases or the number of firms decreases). The queue rationing allocation with price dispersion is determined as follows. As before, let P_i be the price charged by firm i, $i = 1, ..., N_F$. As in (1.3), let $\alpha(P_i, V_{NG})$ be the arrival rate at firm i such that a customer would achieve asset value V_{NG} when the price is P_i, found by solving 1.1 for α.[9]

The queue rationing allocation given the prices will be determined by the intersection between two relations. The first relation arises from the equality between the aggregate number of arrivals at firms and the rate at which customers with the good experience a breakdown of the good (or need to have the service renewed):

$$\mu N_{CG} = \mu(N_C - N_{CNG}) = \sum_{i=1}^{N_F} \alpha(P_i, V_{NG}) \tag{3.2}$$

where N_{CG} is the number of customers with the good and N_{CNG} is the number of customers without the good. Then

$$N_{CNG} = N_C - \sum_{i=1}^{N_F} \alpha(P_i, V_{NG})/\mu \tag{3.3}$$

Since the arrival rate must be lower to yield a higher customer asset value for a given price, N_{CNG} in 3.3 will be an increasing function of V_{NG}.

For the second relation, consider the sum of the expected numbers of customers waiting in firm queues, Q:

$$Q = \sum_{i=1}^{N_F} \frac{\alpha(P_i, V_{NG})}{\sigma - \alpha(P_i, V_{NG})} \tag{3.4}$$

Using 1.3, this simplifies to:

$$\begin{aligned} Q &= \sum_{i=1}^{N_F} \left[\frac{\sigma(\phi - (r+\mu)P_i - rV_{NG})}{(r+\mu)(rV_{NG}+\omega)} - 1 \right] \\ &= N_F \left[\frac{\sigma(\phi - rV_{NG})}{(r+\mu)(rV_{NG}+\omega)} - 1 - \frac{\sigma}{rV_{NG}+\omega} \sum_{i=1}^{N_F} \frac{P_i}{N_F} \right] \end{aligned} \tag{3.5}$$

[9] For some values of P_i and V_{NG}, the arrival rate $\alpha(P_i, V_{NG})$ will not lie between zero and the service rate σ.

Thus Q depends only on the average price and is a decreasing function of V_{NG}. The queue rationing allocation occurs when the number waiting in line, Q, equals the number of customers without the good, N_{CNG}. This determines V_{NG}, which in turn determines the arrival rates at each firm and each firm's profits. In this way, conditions in a queuing market for an arbitrary distribution of prices can be found.

Using the parameter values from Figure 2.1, the two relations determining V_{NG} and the proportion without the good are shown in Figure 3.1, which will later be used to show the effects of increasing dispersion. The downward sloping curve labeled Q arises from 3.5. For a particular distribution of prices (e.g., for dispersion parameter $\delta = .10$, to be defined shortly), the upward sloping curve arises from 3.3. At the intersection, the expected number in the queue, Q, equals the number of customers without the good.

Now consider the effect of increasing dispersion on the equilibrium. Suppose

$$P_i = P(1 - \delta/2) + (i - 1)P\delta/(N_F - 1), i = 1, ..., N_F \tag{3.6}$$

Thus the average price is P and firm prices run from $P(1-\delta/2)$ to $P(1+\delta/2)$. The number of customers in line, Q, depends only on the average price P and therefore the relation between Q and V_{NG} does not shift as δ increases. However, the relation 3.3 shifts upward when dispersion δ increases.[10] Thus greater dispersion yields a lower customer asset value V_{NG} and a higher proportion of customers without the good. Aggregate firm profits decline because some firms are choosing suboptimal prices. However, a firm charging the average price P would experience a greater arrival rate and a higher profit, since $\alpha(P, V_{NG})$ increases as V_{NG} goes down.

The effect of increasing price dispersion is shown in Figure 3.1. The three upward sloping curves show N_{CNG} as a function of V_{NG}, from 3.3. The curve with $\delta = 0$ yields the values of V_{NG} and N_{CNG} in a single price equilibrium. As δ increases to 0.10 and then to 0.15, V_{NG} declines and N_{CNG} increases. Figure 3.2 shows the effect of dispersion on the customer asset value, V_{NG}. Starting from the single price equilibrium, an increase in dispersion initially has no effect on customer asset value (or the number of customers waiting in line). Only when dispersion goes beyond a negligible level does customer asset value begin to fall noticeably.

[10] This can be shown by considering the sum of the pair $\alpha(P_i, V_{NG}) + \alpha(P_j, V_{NG})$, where $j = N_F - i$, and $i < N_F/2$. Then $P_i + P_j = 2P$. It can then be shown that the sum of the pair increases as δ increases, so that $\sum_{i=1}^{N_F} \alpha(P_i, V_{NG})$ increases as δ increases.

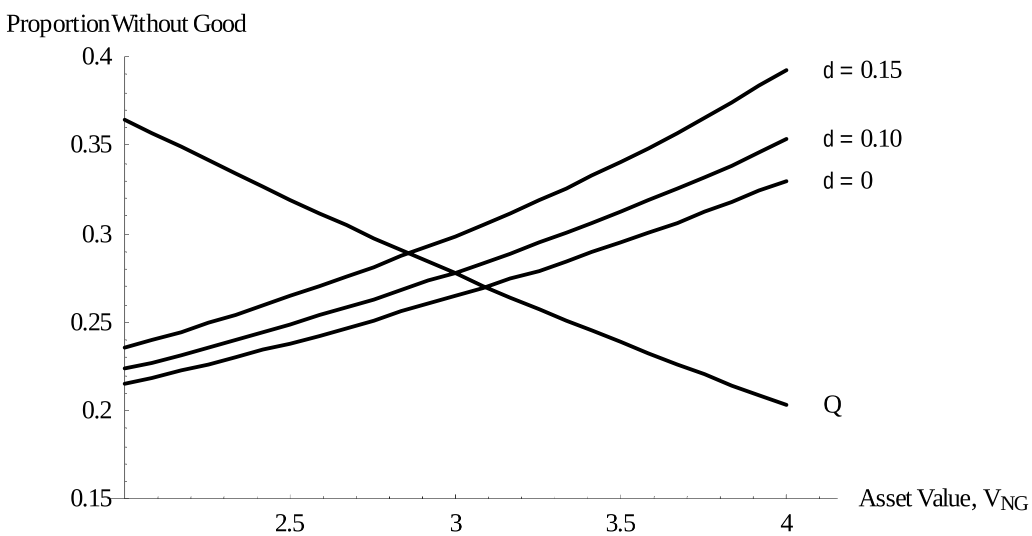

Figure 3.1: Queue Rationing Allocation with Price Dispersion

Price dispersion generates inefficiency in the queuing market.[11] It increases the number of customers waiting in line for a given average arrival rate. Whatever the average price charged by firms, the number of customers waiting in line will be minimized, and the customer asset value will be maximized, at the single price equilibrium. Since the proportion of customers with the good increases, the arrival rate of customers in the market decreases, and production declines when there is price dispersion.

Despite this inefficiency, the behavior of firms and customers in the queuing market is substantially unaffected by price dispersion. Firms continue to face a choice of prices and arrival rates arranged along a customer indifference curve, as they do in the single price equilibrium. Customers face alternative price and arrival rate combinations, unlike the single combination in the single price equilibrium. However, if customers are identical, they are indifferent between the combinations available. The queuing market with price dispersion therefore operates in substantially the same way as in a single price equilibrium. If price dispersion is small, the outcomes are also substantially the same.

4. Conclusions

As observed initially by George Stigler (1961, 1962), goods and labor markets are respectively characterized by price and wage dispersion. The dispersion provides agents with incentives to engage in search. A substantial literature has evolved to explain how price dispersion can arise in the first place, in violation of the Law of One Price. Search theory provides an explanation of how search and matching frictions, with incomplete information, can generate price or wage dispersion in equilibrium (Reinganum, 1979; Burdett and Judd, 1983; Rob, 1985; Bester, 1988). However, simplifying assumptions and necessary conditions for equilibrium impose restrictions on the equilibrium distributions (Butters, 1977; Sattinger, 1991; Burdett and Mortensen, 1998; Arnold, 2000; Mortensen and Wright, 2002).

The queuing model with menu costs provides an alternative explanation of how price dispersion can arise and persist. In the standard supply and demand model with complete and costless information, any differences in price would lead all customers to choose the firm with the lower price. In search models, some customers do not know specific firms with lower prices and optimally choose firms

[11] The inefficiency appears by comparison with the single price queue rationing allocation. Given the menu costs of price changes, inefficiency does not imply that an improvement in the market is possible through elimination of dispersion.

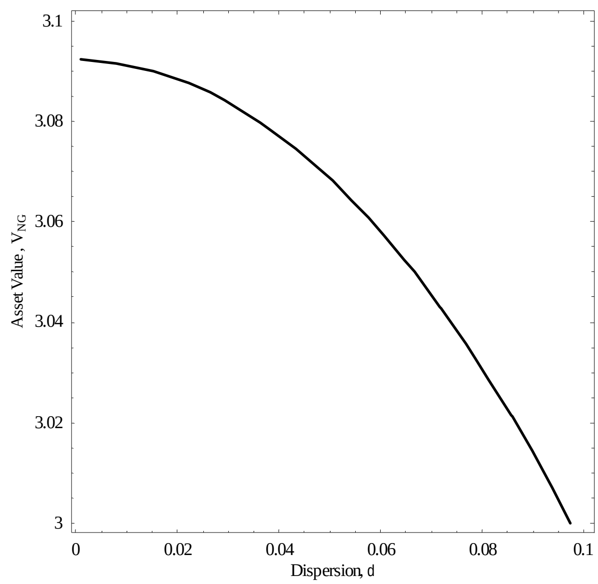

Figure 3.2: Effect of Dispersion on Customers

with higher prices. In the queuing model, optimal customer behavior generates longer expected queues at firms with lower prices, so that firms with higher prices continue to get customers. Customer behavior therefore does not force the elimination of price dispersion. Since queue length (and firm idle time) adjust to price, firms have little profit incentive to adjust suboptimal prices. With menu costs, there will be a range of prices within which firms would not adjust their prices. Price dispersion would then persist in medium and long run equilibrium.

To some extent, elementary queuing phenomena have been incorporated into search theory and matching frictions (e.g., when two workers apply for the same vacancy, causing further delay to one of the workers in finding a job).[12] There is a potential for combining formal queuing and search theories. If customers base choices on actual queue length, stochastic variation in queue lengths provides a natural motivation for search behavior even without price dispersion.

[12] Robert Shimer (2001) incorporates queuing theory in an analysis of the assignment of heterogeneous workers to heterogeneous jobs.

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