Brokers and the Equilibrium Price Function

Michael Sattinger*
Department of Economics
University at Albany
Albany, NY 12222
Email: m.sattinger@albany.edu

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Abstract

This paper describes the equilibrium price function generated by brokers in a market in which heterogeneous buyers meet heterogeneous sellers through a matching process with frictions. The equilibrium price function relates the price to alternative ratios of buyers and sellers offered by different brokers. The paper shows how brokers can enter a matching market and charge fees that yield a profit while making both buyers and sellers better off. Computational methods for deriving the equilibrium price function are developed and the solution is related to the market for access to trading partners.

1. Introduction

This paper describes the equilibrium price function generated by brokers in a market in which heterogeneous buyers meet heterogeneous sellers through a matching process with frictions. Rather than a single price and ratio of buyers to sellers, the solution is characterized by an equilibrium price function relating the price to

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alternative ratios of buyers to sellers that are available on the market. The logic of the solution is that the buyer who values the good the most pays a price that keeps the marginal seller in the market, and the seller with the lowest cost charges a price that keeps the marginal buyer in the market.

The equilibrium price function has previously been described by Mortensen and Wright (2002). This paper shows how brokers perform the function of “market makers” in their paper, develops computational methods for deriving the equilibrium price function for arbitrary distributions of buyers and sellers, and relates the solution to the market for access to trading partners.

Mortensen and Wright, in a paper directed towards conditions under which a single price will prevail in a two-sided search equilibrium model of pure exchange, consider the competitive search equilibrium developed by Moen (1997) and Shimer (1996). They extend the Moen and Shimer results to heterogeneity on both sides of the market and interpret the results as a complete markets Walrasian equilibrium. In the Mortensen and Wright solution, buyers and sellers choose different submarkets in which a single ratio of buyers to sellers prevails. Within a submarket, buyers are identical, sellers are identical, and their indifference curves are tangent, satisfying the Hosios condition for efficiency (Hosios, 1990). They describe the equilibrium price function that assigns buyers and sellers to submarkets (and to each other) through optimal choice of trading ratios.

Mortensen and Wright, attributing the interpretation to Greenwald and Stiglitz (1988), suggest that third party “market makers” with a profit motive could set up the submarkets. This paper shows that competitive profit-maximizing brokers would bring about the conditions for the equilibrium price function. In the absence of brokers, a prevailing wage and trade ratio may result in buyer and seller indifference curves that intersect. A broker, by offering an alternative ratio of buyers to sellers, can charge fees that yield a profit while making both buyers and sellers better off. Section 2 shows how this is possible.

In actual markets, one role of brokers is to facilitate trade, bringing buyers and sellers together. Nti and Shubik (1984) show the existence of equilibrium in a market where all trade goes through designated brokers. Rubenstein and Wolinsky (1987) study a model where prices are negotiated among buyers, sellers and middlemen who meet stochastically. Other work on middlemen includes Johri and Leach (2000), Li (1998), and Shevchenko (2000). Greenwald and Stiglitz (1988) describe a notional employment agency that pays for search and hiring intensities and receives payment for matching, eliminating search externalities. Sattinger (1984, 1990) shows how competitive labor exchanges could also generate efficiency conditions. The role of brokers has also been considered in the context of real estate markets (Gehrig, 1993; Yavaş, 1992, 1994, 1996). Serrano (1995) presents a model in which a player acts as a broker and centralizes trade. Wooders (1997) shows that a broker, in the form of a profit-maximizing monopoly intermediary, sets bid and ask prices that are approximately Walrasian (see also Wooders 1998a, 1998b).

Mortensen and Wright describe a method for computing the equilibrium price function from the steady-state flows of buyers and sellers into the market. This paper derives a procedure for solving for the equilibrium price function from the short run or steady state distributions of buyers and sellers on the market and calculates specific equilibrium price functions. In this procedure, the assignment of buyers to sellers and their trading ratios in each submarket are endogenously determined, along with the endpoints of the equilibrium price function. The procedure emphasizes the role of the second derivative of the equilibrium price function in reconciling distributions of buyers and sellers with the ratios of buyers to sellers in submarkets.

Section 2 describes the behavior of buyers and sellers on the market and the trading frictions they face. Section 3 describes broker behavior and shows the possibility of profits. Section 4 shows how the equilibrium price function can be calculated in the short run. The section also considers changes in buyer and seller participation in the market over time. Section 5 describes the additional conditions that prevail in the steady state. Section 6 relates the results to the market for access to trading partners and discusses features of the solution.

2. Behavior of Buyers and Sellers

2.1. Matching Function

Assume that buyers want to purchase at most one unit of a given good, and sellers want to produce and sell at most one unit of the good at a point in time. Assume that when buyers and sellers attempt to meet to consider a trade in the absence of a broker, the number of meetings is determined by a matching function. Let $M(x, y)$ be the matching function, the number of meetings per period if there are x buyers and y sellers. As in Mortensen and Wright, assume that $M(x, y)$ is increasing in its arguments, concave, and linearly homogeneous (see discussions of the matching function in Mortensen and Pissarides, 1999, and Pissarides, 2000). From the assumption of linear homogeneity, $M(x, y)/y = M(x/y, 1)$. Let θ be the ratio of buyers to sellers and let $m(\theta) = M(\theta, 1)$. In the economic conditions that will be considered here, buyers and sellers trade whenever they meet. The rates at which buyers and sellers get trades are then $m(\theta)/\theta$ and $m(\theta)$, respectively.

2.2. Buyers

Assume that a consumer who has the good receives a constant benefit B per unit of time. The values B vary among atomistic consumers. Suppose the good deteriorates or becomes useless at a rate γ. The flow of asset value of a consumer with the good is then given by B plus γ times the change in asset value if the good deteriorates:

$$rW_{BG} = B + \gamma(W_B - W_{BG}) \tag{2.1}$$

where r is the discount rate, and W_B is the asset value of the consumer without the good. A consumer without the good may decide to enter the market, becoming a buyer. If a buyer uses a broker offering a ratio θ of buyers to sellers and pays a fee F, the asset value flow of the buyer without the good is given by

$$rW_B = \left(\frac{m(\theta)}{\theta}\right)(W_{BG} - W_B - P - F) - c_b \tag{2.2}$$

where c_b is the cost of participating in the market, P is the price of the good, and $m(\theta)/\theta$ is the number of matches per buyer per unit of time. The flow of asset value for a buyer is then given by the rate at which a seller is found times the increase in asset value minus the payment of the price and fee, minus the cost of participating in the market. Solving 2.1 and 2.2 for W_B yields

$$W_B(B, \theta, P + F) = \frac{(m(\theta)/\theta)[B - (r + \gamma)(P + F)) - (r + \gamma)]}{r(r + \gamma + m(\theta)/\theta)} \tag{2.3}$$

2.3. Sellers

On the seller side, let S be the seller parameter and suppose S varies among atomistic sellers. Assume a seller can only sell one good at a time, and can reenter the market immediately after selling the good, with the same parameter S. Suppose the asset value for a seller with parameter S who chooses to enter the market is

$$W_S(S, \theta, P + F) = (m(\theta)(P + F - S) - c_s)/r \tag{2.4}$$

where c_s is the cost per unit of time to the seller from participating in the market, P is the price and F is the fee received by the seller if a sale occurs at a broker offering ratio θ. The seller's asset value is the present value of the likelihood of a trade per unit of time times the profit from a trade, minus the cost of participating. The seller parameter S may be regarded as the cost of the good to a particular seller. If the maximum expected value of $W_S(S, \theta, P + F)$ is less than zero, the seller would choose not to enter the market. Mortensen and Wright show that if the interest rate is sufficiently high, trades between buyers and sellers would not take place.

3. Brokers

Assume brokers can combine buyers and sellers and generate trades using the same matching technology available to buyers and sellers in the absence of brokers. Assume brokers incur no costs by doing so (costly broker activity, or alternative matching technologies, can be considered separately).

Brokers attempt to make profits by charging more for buyers than they pay sellers (or paying sellers less than they charge buyers). If a buyer indifference curve goes through a particular point, a broker could offer that buyer a combination of fee and θ that lies on the indifference curve. Similarly, if a seller indifference curve goes through a point, a broker could offer that seller a combination of fee and θ that lies on the seller's indifference curve. If a buyer's indifference curve lies above a seller's indifference curve at a particular ratio θ, brokers face a profit opportunity. Competition among brokers eliminates all profit opportunities, so that the fee charged to buyers cannot exceed the fee paid to sellers. Since brokers would drop out if the fee charged to buyers were less than the fee paid to sellers, the fees must be equal and broker profits are eliminated by entry. As a result of broker activity, the buyer and seller indifference curves are tangent at the fee charged by brokers and ratio of buyers to sellers.

The effect of broker activity with identical buyers and sellers is shown in Figure 3.1. With no broker and with a given price P, the buyer and seller indifference curves go through the point with $F = 0$ at the ratio of buyers to sellers. Since the indifference curves intersect, there are profit opportunities for brokers at a lower ratio of buyers to sellers. When brokers enter at the lower ratio, the ratio of buyers to sellers that do not go through the broker is greater than the original ratio. Competition among brokers moves the fees charged to buyers and paid to sellers closer together, attracting more buyers and sellers to the brokers. The ratio θ offered by brokers moves to the original level as buyers and sellers switch to brokers, and the fee moves to 1, at which the buyer and seller indifference curves

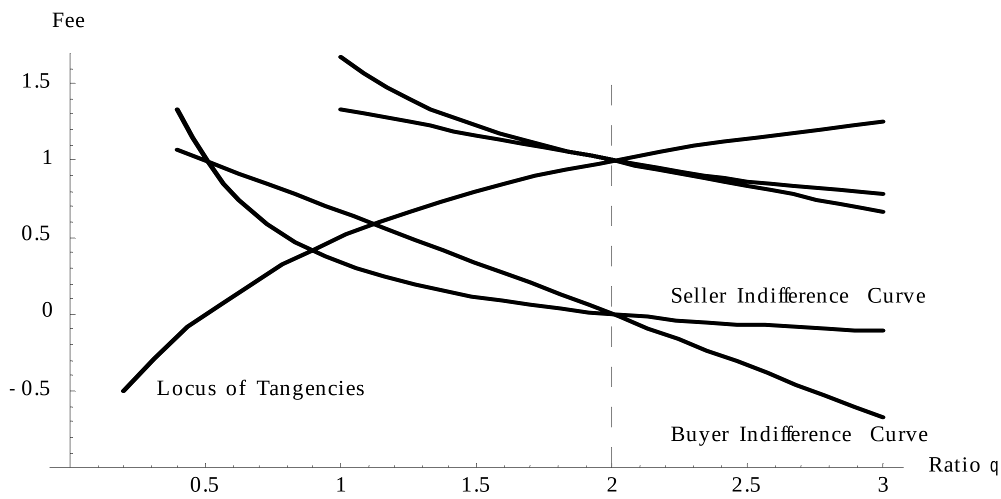

Figure 3.1: Effects of Broker Activity

are tangent.[1]

The effect of the broker activity is that buyers and sellers have the same trade-offs between the ratio θ and the common fee, F, thereby implementing a market for access to trading partners. The condition assures that there are no externalities from entry of buyers or sellers, as arise in the literature on search congestion. In this solution, there is no incentive to bargain or engage in search.[2]

It can be argued that brokers are not needed to bring about the equilibrium shown in Figure 3.1, since buyers and sellers in a market can ordinarily be expected to reach a point on a contract curve where indifference curves are tangent. However, in this context, individual buyers and sellers cannot in general offer a ratio of buyers to sellers.[3] Further, interacting with sellers to negotiate price and ratio is probably more difficult than finding a seller. Brokers, in dealing with many buyers and sellers, can more reasonably offer a ratio of buyers to sellers. The distinction between price and fee is relevant to understanding the role of brokers. However, the price and fee enter the buyer and seller asset equations in

[1] This diagram assumes the ratio of buyers to sellers stays the same as the fee changes.

[2] The consequences of tangent indifference curves for efficiency are discussed by Mortensen and Wright (1997). See also Pissarides (2000) and Mortensen and Pissarides (1999).

[3] An exception occurs in labor markets, where an employer may be able to attract a number of applicants to a vacancy.

the same way, so only the sum, $P + F$, is determined by equilibrium conditions. In the analysis that follows, let p be the sum of the price and fee determined by broker activity.

Now consider the role of brokers when buyers and sellers are heterogeneous. First consider the marginal rates of substitution for buyers and sellers. From 2.3, with $p = P + F$, setting $W_B(B, \theta, p)$ equal to a constant and solving for p yields an indifference curve for a particular buyer parameter B. Let $MRS_B(B, \theta, p)$ be the buyer's marginal rate of substitution of the price for the ratio. It is the absolute value of the slope of the indifference curve at price p and ratio θ for a buyer with parameter B. Setting $MRS_B(B, \theta, p)$ equal to a given slope of the price function, p', and solving for B yields the buyer parameter as a function of θ, p and p'; let this function be given by $b(\theta, p, p')$. The function $b(\theta, p, p')$ determines the buyer parameter B for the buyer who would choose the broker offering total price p and ratio θ when the rate of change of the price is p', assuming the buyer second order condition is satisfied.[4] As with a buyer, setting $W_S(S, \theta, p)$ equal to a constant and solving for p generates an indifference curve for a seller. Let $MRS_S(S, \theta, p)$ be the seller's marginal rate of substitution of the price for the ratio. Let $s(\theta, p, p')$ be the seller parameter obtained by setting $MRS_S(S, \theta, p)$ equal to a particular slope p' and solving for S, again assuming the second order condition is satisfied.

At a particular combination of price and trading ratio, the slopes of the indifference curves for different buyers will vary, and likewise the slopes of the indifference curves for different sellers will vary. The marginal rate of substitution for buyers, $MRS_B(B, \theta, p)$, is an increasing function of the buyer parameter B. Through a particular point, buyers with higher values of B will have steeper indifference curves. The marginal rate of substitution for sellers, $MRS_S(S, \theta, p)$, is a decreasing function of the seller parameter S, and sellers with higher values of S will have flatter indifference curves.

A broker can profitably enter whenever the price some buyers are willing to pay exceeds the fee some sellers are willing to accept at a particular ratio. As a result, an equilibrium in this market cannot occur with a single price and trading ratio, since at either a higher or lower ratio, a buyer indifference curve will go above a seller indifference curve. The broker can always get rid of buyers or sellers to yield a particular ratio, so the level of the ratio does not prevent the broker from making a profit. Entry of costless brokers and competition imply that at a particular ratio in any equilibrium, the price that buyers are willing to pay can be no larger than

[4] The function $b(\theta, p, p')$ is analytic, so the value of B can always be found for given values of θ, p and p'. However, there may be no buyer in the market with such a value of B.

the fee sellers are willing to accept. For this to be the case, no part of any buyer's indifference curve can lie above any seller's indifference curve. This condition will obtain if buyers and sellers face a common price curve (the price p as a function of the trading ratio θ), with buyers choosing points of tangency below the curve and sellers choosing points of tangency above the curve.

4. Short Run Equilibrium Price Function

The equilibrium price function, generated by competitive brokers, can be found by solving the differential equation arising from the density functions of buyers and sellers. The endpoints of the equilibrium price function can be found from conditions that will be discussed separately.

Mortensen and Wright describe a different procedure for determining the equilibrium price function. They begin with the steady-state flows of buyers and sellers into the market at each buyer and seller parameter value, respectively. From the value function for buyers and sellers, buyers with higher valuation of the good will be assigned to sellers with higher costs. The relation between buyer and seller parameters can then be determined from the flows. The parameter of the seller choosing a particular ratio θ depends on θ and the price at θ. Substituting this expression for the seller parameter into the condition for tangency between the buyer and seller indifference curves at θ yields a first order differential equation for the price. The endpoints and the initial (boundary) conditions for the price function are determined from the condition that the marginal buyers and sellers (with the lowest value of the good and highest cost, respectively) must have values of participation of zero in the steady-state solution. The densities of buyers and sellers in a particular submarket with ratio θ could then be found from the flows of buyers and sellers at θ and the match rate at θ.

In contrast, the solution method proposed here starts with arbitrary distributions of buyers and sellers on the market. The assignment of buyers to sellers is then determined endogenously, together with the price function and endpoints. The distribution of seller parameters on the market takes a different form, since sellers return to the market immediately after selling a good. Since the buyers and sellers choosing a particular ratio θ depend on the slope of the price function as well as the price, a second order differential equation arises for the price function. This section also shows how the procedures can be applied to specific functional forms.

Let $f(B)$ be the density function for buyer parameters, so that there are

$$\int_{B1}^{B2} f(x)dx \tag{4.1}$$

buyers with parameter values between $B1$ and $B2$. Let $g(S)$ be the density function for seller parameters, so that there are

$$\int_{S1}^{S2} g(x)dx \tag{4.2}$$

sellers with parameter values between $S1$ and $S2$. Suppose at a particular ratio θ of buyers to sellers, the price is p and the rate of change of the price with respect to θ is p'. Then the parameter for the buyer choosing θ is $b(\theta, p, p')$ and the parameter for the seller choosing θ is $s(\theta, p, p')$. The functions f and g provide the densities with respect to the buyer and seller parameters respectively. To find the ratio of buyers to sellers at a particular point, it is necessary to have the densities with respect to a common variable. This is accomplished by multiplying the functions f and g by the appropriate Jacobians, given by the total derivatives of the buyer and seller parameters with respect to the ratio θ. The density of buyers choosing θ is then

$$f(b(\theta, p, p'))\frac{db(\theta, p, p')}{d\theta} \tag{4.3}$$

where

$$\frac{db(\theta, p, p')}{d\theta} = \frac{\partial b}{\partial \theta} + \frac{\partial b}{\partial p}\frac{\partial p}{\partial \theta} + \frac{\partial b}{\partial p'}\frac{\partial p'}{\partial \theta} = \frac{\partial b}{\partial \theta} + \frac{\partial b}{\partial p}p' + \frac{\partial b}{\partial p'}p'' \tag{4.4}$$

In this expression, p'' is the second derivative of the price with respect to the ratio θ and measures the curvature. The density of sellers choosing θ is

$$g(s(\theta, p, p'))\frac{ds(\theta, p, p')}{d\theta} = g(s(\theta, p, p'))\left(\frac{\partial s}{\partial \theta} + \frac{\partial s}{\partial p}p' + \frac{\partial s}{\partial p'}p''\right) \tag{4.5}$$

The ratio of densities at θ must equal the ratio of buyers to sellers:

$$\frac{f(b(\theta, p, p')db(\theta, p, p')/d\theta}{g(s(\theta, p, p'))ds(\theta, p, p')/d\theta} = \theta \tag{4.6}$$

Solving 4.6 for p'' as a function of p and θ yields a second order nonlinear differential equation. If the density of buyers is very low, the second derivative p'' will approach the second derivative of the seller indifference curve at θ, so that

$b(\theta, p, p')$ will change relatively more than $s(\theta, p, p')$ as θ changes. If instead the density of sellers is very low, the second derivative p'' will approach the second derivative of the buyer indifference curve at θ, so that $s(\theta, p, p')$ will change relatively more than $b(\theta, p, p')$ as θ changes. In this way, the second derivative p'' reconciles the densities at θ with the ratio θ itself. With both densities positive, the second derivative p'' will lie between the second derivatives of the buyer and seller indifference curves at θ, guaranteeing that the buyer and seller second order conditions are satisfied.

The price function satisfying 4.6 will have endpoints determined by the condition that no brokers could profitably offer lower or higher trading ratios. Suppose θ_L is the lowest ratio at which buyers and sellers are currently trading through a broker. Suppose a buyer with parameter $B1$ and seller with parameter $S1$ have indifference curves that are tangent to the price function at θ_L. If there are buyers with higher parameters than $B1$ or sellers with higher parameters than $S1$, their indifference curves would cross at θ_L and a broker could profitably offer buyers and sellers opportunities to trade at a lower θ. However, if the buyer at θ_L has the maximum parameter $B_{\max}$ and the seller has the maximum parameter $S_{\max}$, no trades at ratios lower than θ_L would be profitable and θ_L would be the lower endpoint. Similarly, the highest trading ratio θ_H occurs at a point of tangency between the buyers and sellers with the lowest parameters, $B_{\min}$ and $S_{\min}$.

In the short run, the maximum seller parameter $S_{\max}$ and the minimum buyer parameter $B_{\min}$ are determined by conditions in the previous period when entry decisions are made. On the basis of conditions in the previous period, the seller with the maximum parameter $S_{\max}$ expected an asset value equal to zero and was indifferent between entering and staying out of the market. However, the current conditions may yield a different (and possibly negative) asset value.[5] Similarly, $B_{\min}$ is determined by conditions in the previous period, and the asset value for $B_{\min}$ under current conditions may not be zero.

Unfortunately, knowing the values of $B_{\max}$ and $S_{\min}$ does not by itself determine the lowest trading ratio. The reason is that the locus of tangencies between the buyer's and seller's indifference curves extend over different ratios and prices. The locus of tangencies is found by setting $MRS_B(B, \theta, p)$ equal to $MRS_S(S, \theta, p)$ and solving for the price. Let $p_{\tan}(B, S, \theta)$ be the price at ratio θ such that the indifference curve for the buyer with parameter B is tangent to the indifference

[5] With participating assets c_s sunk, the marginal seller would only drop out if $P + F - V_{s\max}$ fell below zero.

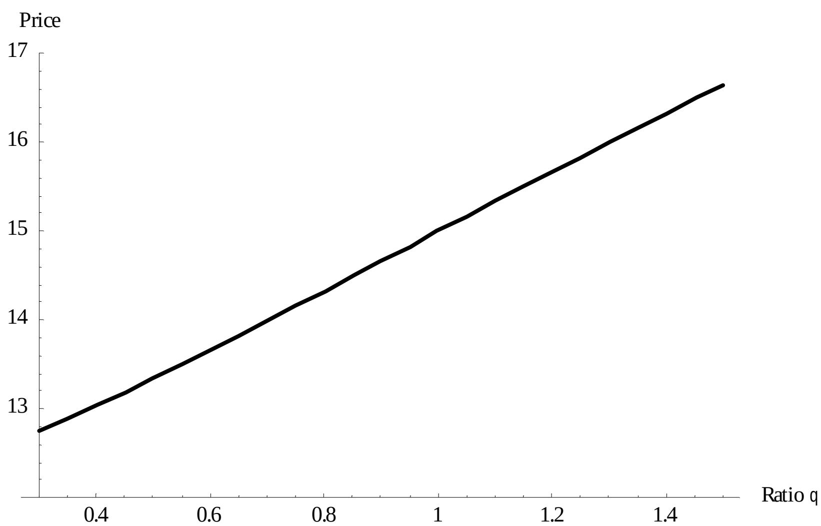

Figure 4.1: Condition for Left Endpoint

curve for the seller with parameter S.[6] Figure 4.1 shows $p_{\tan}(B_{\max}, S_{\max}, \theta)$.[7] The endpoint for the lowest trading ratio lies somewhere along this curve, but the specific location depends on other conditions in the market. A similar locus of tangencies restricts the location of the endpoint with the highest ratio.

Figure 4.2 shows a short run equilibrium price function for given density functions and maximum and minimum buyer and seller parameter values. Figure 4.3 shows the same equilibrium price function together with the indifference curves for the buyers and sellers at the endpoints. In practice, the endpoints are found by choosing a trial value of θ_L. The price is then $p_{\tan}(B_{\max}, S_{\max}, \theta_L)$ and the slope of the price function can be found from either $MRS_B(B_{\max}, \theta_L, p_{\tan}(B_{\max}, S_{\max}, \theta_L))$ or $MRS_S(B_{\max}, \theta_L, p_{\tan}(B_{\max}, S_{\max}, \theta_L))$. The price and slope at θ_L determine the

[6] The function $p_{\tan}(B, S, \theta)$ is given by

$$\frac{c_b + B}{r + \gamma} + \left(\frac{\theta(1+\theta)(1+(r+\gamma)(1+\theta))((r+\gamma)S - c_b}{(r+\gamma)(1+(r+\gamma)(1+\theta)^2)m(\theta)} - B\right)m'(\theta)$$

[7] The parameter values for the short run solution are $r = .1$, $\gamma = .05$, $c_b = .2$, $c_s = .4$, $p = 12$ in the absence of brokers, $B_{\max} = 4$, $B_{\min} = 2$, $S_{\max} = 12$ and $S_{\min} = 10$. The density functions for buyers and sellers are $f(B) = (4 - B)(2/3)$, $B \leq 4$ and $g(S) = (S - 6)(1/6)$, $S \geq 6$. The matching function is assumed to be $m(\theta) = \theta/(1 + \theta)$.

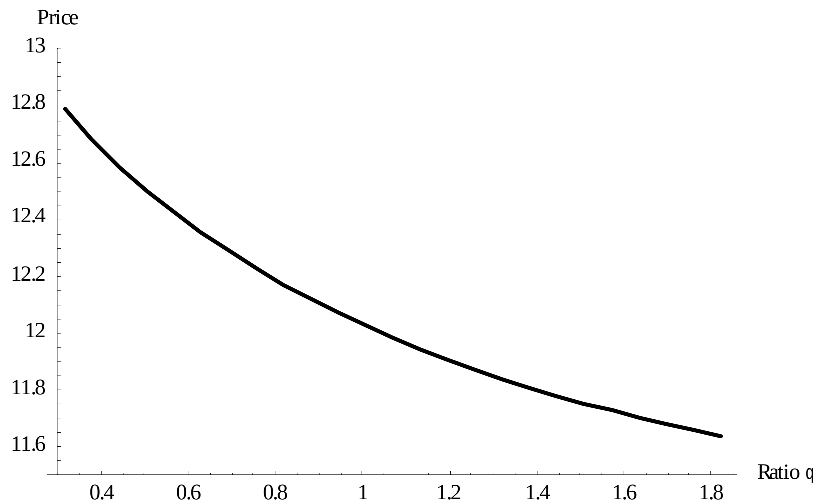

Figure 4.2: Short Run Fee Function

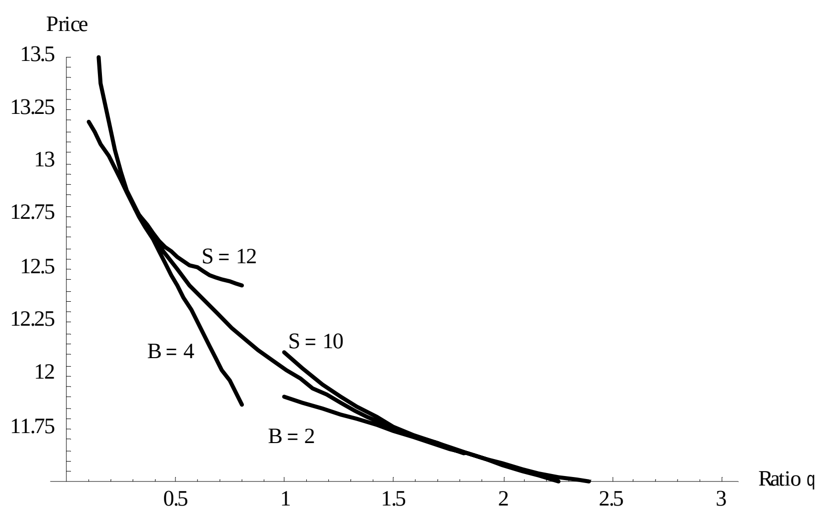

Figure 4.3: Short Run Price Functions with Indifference Curves at Endpoints

initial conditions for the differential equation obtained from 4.6, which can then be solved numerically. If the trial value of θ_L yields a value of S at $B = B_{\min}$ that exceeds $S_{\min}$, the trial ratio θ_L is lowered until $S = S_{\min}$ when $B = B_{\min}$.

In the short run solution shown in Figures 4.2 and 4.3, the left endpoint is at $\theta = .315$ and $p = 12.79$, and the right endpoint is at $\theta = 1.82$ and $p = 11.64$. At the left endpoint, $W_B(4, .315, 12.79) = 1.71$ and $W_S(12, .315, 12.79) = -.21$. At the right endpoint, $W_B(2, 1.82, 11.64) = .12$ and $W_S(10, 1.82, 11.64) = .65$.

In general, the existence and uniqueness of the equilibrium price function arise from the existence and uniqueness of the differential equation through an initial point. However, in this case the initial point used in the determination of the differential equation cannot be specified a priori. Instead of initial conditions, the differential equation satisfies boundary conditions requiring that the initial and end points satisfy the endpoint conditions.

Dynamic adjustment in the short run arises through entry of marginal buyers and sellers and changes in the densities of buyers and sellers. In the next period, buyers at lower parameter values enter when the buyer with the current minimum parameter $B_{\min}$ experiences a positive asset value. For example, in the short run solution above, some marginal buyers with $B < 2$ will enter. With expectations based on current market conditions, the marginal buyer in the next period has a parameter value that yields a zero asset value at the current lowest trading ratio, θ_L, and price corresponding to that ratio. The maximum parameter $B_{\max}$ will remain the same from one period to another if there are always new entrants with $B_{\max}$. The density of buyers may also change because unmatched buyers in the current period will be added to new entrants in the next period. Dynamic adjustment for the marginal seller and the density of sellers can be described analogously. In the short run solution above, some marginal sellers with $S < 12$ will leave. A feature of a short run equilibrium that is not a long run equilibrium is that marginal buyers and sellers can regret entry, experiencing conditions that would have made them better off staying out of the market. Also, potential buyers and sellers who stay out of the market can regret not entering.

5. Steady State Equilibrium Price Function

In the long run steady state equilibrium, additional conditions are imposed. The marginal buyer and seller are determined endogenously instead of from previous conditions. The marginal buyer, with parameter $B_{\min}$, would get a zero asset value from the steady state equilibrium price function, so the value of $B_{\min}$ would not change. Similarly, $S_{\max}$ will yield a zero asset value for the marginal seller. The density function for buyers and sellers will satisfy stock-flow conditions consistent with unchanging densities over time. For buyers, the stock-flow conditions require that at each parameter value, the flow of buyers getting the good from matches equals the flow of new buyers into the market. An analogous condition holds for sellers.

From the condition that the asset value for the marginal seller will be zero in the long run equilibrium, it is possible to find the new endpoint condition by solving

$$W_S(S_{\max}, \theta, p_{\tan}(B_{\max}, S_{\max}, \theta)) = 0 \tag{5.1}$$

for $S_{\max}$. The result yields $S_{\max}$ as a function of θ. Substituting this function into $p_{\tan}(B_{\max}, S_{\max}, \theta)$ yields the price at the lowest ratio as a function of the lowest ratio.

Figure 5.1 shows the steady state equilibrium price function under the assumption that the density of buyers in the market takes a particular form.[8] Figure 5.2 shows the same equilibrium fee function together with the indifference curves for buyers and sellers with the highest and lowest parameter values. The marginal seller has $S_{\max} = 11.76$, compared to $S_{\max} = 12$ in the short run solution. The marginal buyer has $B_{\min} = 1.87$, compared to 2 in the short run solution.

In general, the density of buyers in the market will be determined endogenously for a given density of all consumers, with and without the good. However, given the density of buyers in the market, it is possible to determine the distribution of all consumers that would yield the steady state equilibrium shown in Figures 5.1 and 5.2.

Consider a particular trading ratio, θ. The buyer choosing this ratio has parameter $b(\theta, p(\theta), p'(\theta))$, where $p(\theta)$ and $p'(\theta)$ are the price and slope at θ determined by the steady state equilibrium price function. At θ, the flow of buyers with parameter $b(\theta, p(\theta), p'(\theta))$ who get the good is $(m(\theta)/\theta) f(b(\theta, p(\theta), p'(\theta))$. Let $f_{total}(B)$ be the density of all consumers with parameter value B, whether or not they have the good or are in the market. The flow of buyers into the market (if $B \geq B_{\min}$) is $\gamma(f_{total}(B) - f(B))$. In the steady state, the flow of buyers getting the good must equal the flow of consumers entering the market at each parameter value. Solving

[8] The parameters and matching function for this long run solution are the same as for the short run solution. It is also assumed that $f(B) = (4 - B)(2/3)$, $B \leq 4$, the same as in the short run solution (the density of sellers is also the same). The left endpoint is at $\theta_L = .466$ and $p = 13.02$. The maximum seller parameter is 11.76. The right endpoint is at $\theta_R = 2.53$ and $p = 11.79$. The minimum buyer parameter is $B = 1.87$.

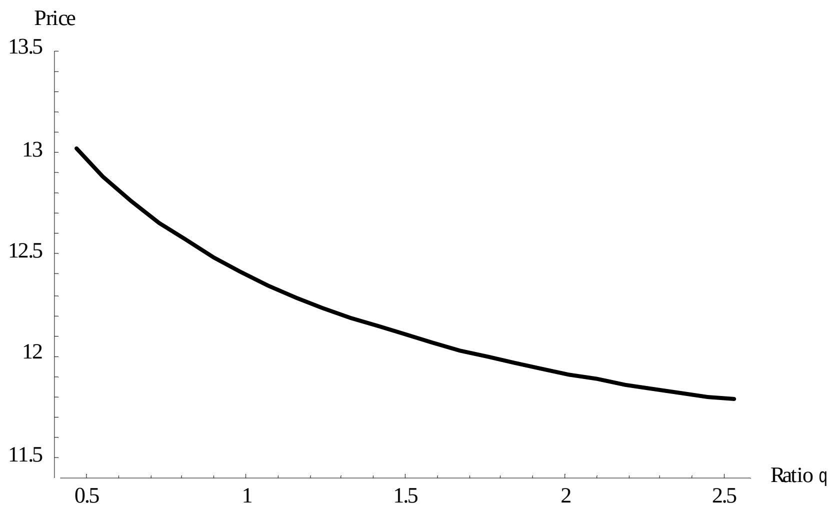

Figure 5.1: Steady State Equilibrium Price Function

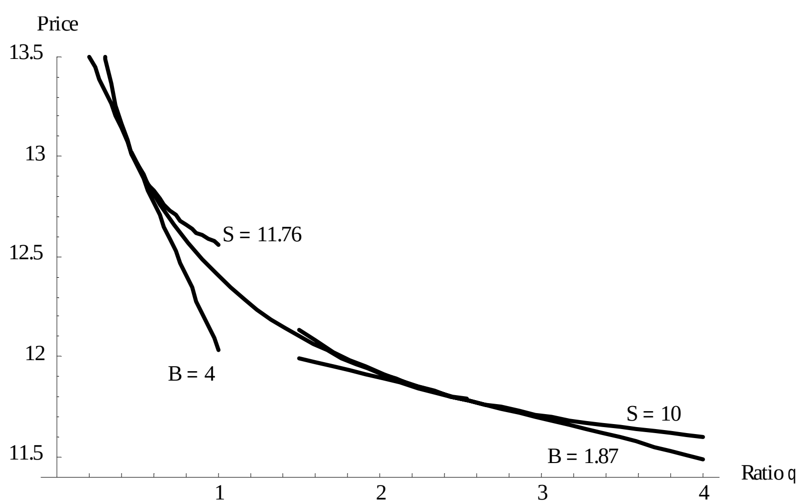

Figure 5.2: Steady State Equilibrium Price Function with Endpoint Indifference Curves

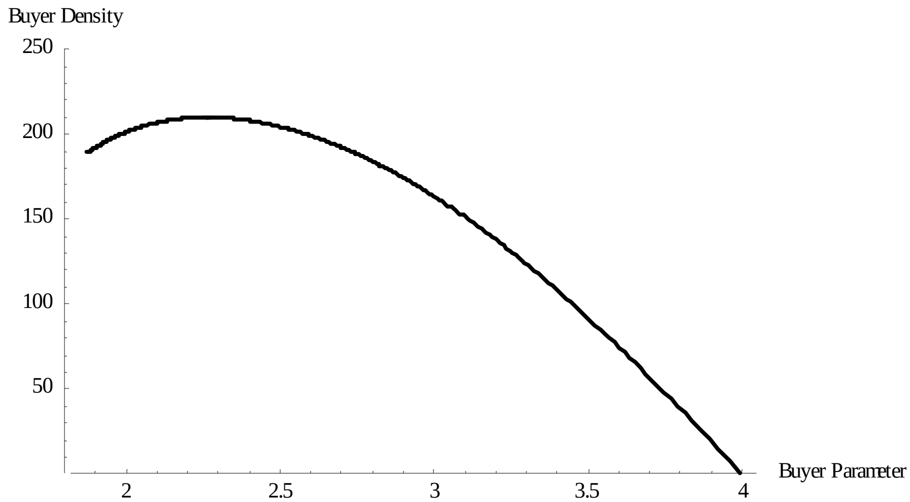

Figure 5.3: Density of All Buyers

yields

$$f_{total}(b(\theta, p(\theta), p'\theta)) = \left(\frac{\gamma + m(\theta)/\theta}{\gamma}\right) f(b(\theta, p(\theta), p'\theta)), \theta_L \leq \theta \leq \theta_H \qquad (5.2)$$

By varying θ, the function f_{total} can be found parametrically. Figure 5.3 shows the density function for all consumers consistent with the steady state equilibrium shown in Figure 5.1. By construction, the distribution of all sellers is the same as the short run distribution, since a seller that engages in a trade immediately reenters the market with the same parameter value.

6. Conclusions

As a solution to the problem of trading among heterogeneous agents, the short and long run equilibrium price functions present several problems. The first is computational. The complexity of the second order differential equation rules out analytical solutions. Also, the endpoints of the price function cannot be found without obtaining the equilibrium price function. Use of numerical methods prevents an explicit solution for the time path of dynamic adjustments, as can be derived in the homogeneous agent case (Sattinger, 2000).

A second problem with the solution arises from the abilities required of agents in the heterogeneous agent solution. Buyers and sellers face difficulty in finding trading partners, resulting in limitations on trades as reflected in the matching function. However, in the heterogeneous agent solution, buyers and sellers are assumed to be able to find the brokers with ratios that maximize their asset function. It is somewhat inconsistent to expect agents to be able to find brokers with certainty but not trading partners. The solution also requires an infinite number of brokers in order to provide markets for all potential traders.

Mortensen and Wright have proposed that costs of brokers (market makers) would reduce the number of brokers. With sufficiently high broker costs, the number of ratios would be reduced to the upper and lower levels (or perhaps a single level), and explicit analytic solutions over time could be found. Combined with broker costs that limit the number of ratios offered to two, brokers could offer lotteries to buyers and sellers. At a particular intermediate ratio, θ, the broker could offer a buyer a probability of a trade at the higher ratio and one minus that probability of a trade at the lower ratio. The probabilities of trade at the two ratios would be such that the lottery yields the buyer a likelihood of trade per unit of time of $m(\theta)/\theta$. The fee charged by the broker would be the actuarially fair fee. The actuarially fair fee as a function of the intermediate ratio θ chosen by a buyer would be a curve running from the one ratio-fee combination to the other. Another lottery could be offered by the broker to sellers. The lotteries extend the options available to buyers and sellers if there are only two trading ratios. However, the buyer fee lottery curve can be shown to lie above the seller fee lottery curve. Although a costless broker could enter and make profits at intermediate ratios, broker costs could prevent entry at intermediate levels.

The homogeneous agent solution, reflected in Figure 2.1, may be regarded as a limiting case of the heterogeneous agent solution. Mortensen and Wright show that if $r = 0$ in their model, the efficient price function reduces to a single point, so that there is only one price and ratio in steady state equilibrium. Further, as r approaches zero, the endpoints approach each other. These results support the use of the homogeneous agent solution to price dynamics as an approximation to the heterogeneous agent solution.

The equilibrium price function can be related to the market for access to trading partners, which arises when agents can pay something to increase their chances of a trade. In a market with frictions or in disequilibrium, finding a trading partner has value. The absence of a market for access to trading partners generates inefficiencies and congestion externalities. Mechanisms that generate a market for access to trading partners satisfy the Hosios condition and result in efficiency. Mechanisms include a market for interviews (Sattinger, 1990, 1995), competitive search equilibrium (Moen, 1997, Shimer 1996, Mortensen and Wright, 2002) and queuing (Sattinger, 2002).

In the markets considered here, a market for access to trading partners arises because a buyer can increase his or her chances for finding a trading partner by choosing a broker with a lower ratio of buyers to sellers, at the cost of a higher combined price and fee. Sellers can similarly achieve a greater likelihood of trading by choosing a broker with a higher ratio of buyers to sellers. In equilibrium, buyers and sellers at the same ratio have equal trade-offs between the price and the ratio of buyers to sellers. The equal trade-offs arise because the indifference curves of buyers and sellers are tangent to the same equilibrium price function, satisfying the Hosios condition at each ratio.

The unusual feature of the equilibrium price function solution is that the trade-offs are unequal among buyers. When the discount rate is positive, buyers in equilibrium are choosing brokers at which the amount they are willing to pay for an increase in match probability is unequal. Buyers with higher parameter B are willing to pay more for such an increment than buyers with low values of B. A similar dispersion in trade-offs holds on the seller side. The trade-off differences yield a potential for further trade.[9] Arbitrage in match probabilities among buyers and among sellers would eliminate the dispersion in trade-offs, but there is no mechanism in the model that would facilitate such arbitrage.

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[9] Another unusual feature of the equilibrium price function solution is that the total number of trades per unit of time is not maximized since trading occurs at different ratios of buyers to sellers.

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