The Holiday Puzzle Solution - Groundhog Day, 2014

THIS YEAR: ALBANYHENGE!

On December 21, the sun is overhead at the Tropic of Capricorn (latitude 23.4° S). Someone would like to build columns in Albany (latitude 42.7° N) so that the columns are in a line which points to the direction of sunrise on December 21. Carefully describe this direction (with justification)! You may make simplifying assumptions such as ignoring atmospheric refraction.

SOLUTION: The direction is approximately 32.7° south of due east.

To see this, suppose the $xy$ plane is tangent to a spherical Earth at Albany with the $x$-axis pointing due east and the $y$-axis pointing due north. Let the $z$-axis point straight up. At sunrise, the direction of the sun is described by $(\cos \theta, -\sin \theta, 0)$ where $\theta$ is the angle telling us how far south of east the sun is.

Now consider rotating about the $x$-axis so that the $y$-axis is rotated towards the $z$-axis by an angle $\alpha$ which equals the latitude of Albany. (See separate figure.) If the new coordinates are $(x', y', z')$, then $x' = x$, $y' = z \sin \alpha + y \cos \alpha$, and $z' = z \cos \alpha - y \sin \alpha$. In these coordinates, the direction of the sun at sunrise is $(\cos \beta, -\sin \beta \cos \alpha, \sin \beta \sin \alpha)$. If the sun is located overhead at $\gamma$ degrees south of the equator, then $-\sin \gamma = -\sin \theta \cos \alpha$. So $\sin \beta = \sin \gamma / \cos \alpha$ and $\beta = \sin^{-1}(\sin \gamma / \cos \alpha)$. By computation, $\beta = 32.7°$ if $\gamma = 23.4°$ and $\alpha = 42.7°$.

Those familiar with the University at Albany campus may start to wonder if the architect Edward Durell Stone made the Academic Podium his own personal "Stone-henge". Looking at a Google map of the campus, I estimated that the long direction of the Academic Podium points approximately 38° degrees south of east. So the direction does not perfectly line up with the sunrise at the winter solstice. However, if you wait a little while after sunrise then, the sun will cast its light well along the hallways of the Earth Science building and for quite aways on the walkway on the “south” side of the Earth Science building. Indeed, seeing this lighting before an 8 AM exam just before the winter solstice inspired me to ask this question.

No one submitted a solution although there were some comments.

I hope to have another Holiday Puzzle next year. Look for it at http://www.albany.edu/~martinha/puzzle.html

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