1. a) Classify all groups of order $2^{r+1}$ containing an element of order $2^r$. (Hint: Any such group is an extension of $\mathbb{Z}_{2^r}$ by $\mathbb{Z}_2$, but you must also show that the extensions you obtain are not isomorphic to one another.)

b) When $r = 3$, which such group corresponds to the subgroup $L$ of Problem Set 1?

2. Classify the groups of order 18. (Hint: How many conjugacy classes of elements of order 2 are there in $\text{GL}_2(\mathbb{Z}_3)$?)

3. a) Classify all groups of order 12 with a normal 3-Sylow subgroup.

b) In the classification list, which groups are isomorphic to $Q_{12}$, $D_{12}$ and $D_6 \times \mathbb{Z}_2$?

4. Let $G$ be a group of order 60 whose 5-Sylow subgroups are not normal.

a) Show that $G$ is simple. (Hint: First calculate the orders of the normalizers of the 5-Sylow and 3-Sylow subgroups of $G$ and then go through all possible orders for the subgroups of $G$ and show that no subgroup of that order can be normal. In some cases it is possible to show that no subgroup of the stated order can exist at all. The theorems about groups whose order divides 60 given in the text will be useful.)

b) Show that $G$ is isomorphic to $A_5$. (Hint: Find a subgroup of index 5. It may help to calculate the order of the centralizer of an element of order 2 in $G$.)