1. Find a group $G$ and subgroups $H \subset K$ with $H \triangleleft K$, $K \triangleleft G$ but $H$ not normal in $G$.

2. a) For $g \in G$ with centralizer $C_G(g)$, show that $C_G(g) \subset N_G(\langle g \rangle)$.

   b) Find an example of a group $G$ and an element $g \in G$ with $C_G(g) \neq N_G(\langle g \rangle)$.

3. Let

   $$H = \{e, (1 \ 2)(3 \ 4), (1 \ 3)(2 \ 4), (1 \ 4)(2 \ 3)\} \subset A_4$$

   a) Show that $H \triangleleft A_4$.
   b) Show that $H \triangleleft S_4$.
   c) Show that $S_4/H \cong S_3$.
   d) Find a subgroup $K$ of order 8 in $S_4$ containing $H$, and construct an isomorphism from a familiar group of order 8 to $K$.

4. Show that $A_4$ has no subgroup of order 6. Deduce that $A_4$ is the only subgroup of $S_4$ of order 12.