1. Let \( P = e_1, \ Q = \cos \frac{2\pi}{3} e_1 + \sin \frac{2\pi}{3} e_2, \ R = \cos \frac{4\pi}{3} e_1 + \sin \frac{4\pi}{3} e_2, \) and \( N = e_3. \)

Let
\[
\begin{align*}
  u(t) &= \cos t N + \sin t P, \\
  v(t) &= \cos t N + \sin t Q, \\
  w(t) &= \cos t N + \sin t R,
\end{align*}
\]
with \( 0 < t < \frac{\pi}{2} \). Let
\[
\Delta(t) = \Delta u(t)v(t)w(t),
\]
the spherical triangle whose vertices are \( u(t), \ v(t) \) and \( w(t) \).

(a) Use a computer algebra system to compute the angle sum in \( \Delta(t) \).

(b) Show that as \( t \) varies, the angle sum can take on any value in \((\pi, 3\pi)\) (use calculus).

(c) What is the side length of \( \Delta(t) \)?

2. We previously defined rotations in \( \mathbb{R}^3 \) about points in \( S^2 \). Here, for \( 0 \neq v \in \mathbb{R}^3 \) we let \( \rho_{(v, \theta)} \) denote the rotation by \( \theta \) about \( \frac{v}{||v||} \). Note that to find its matrix you do need to normalize \( v \) as part of finding a suitable orthonormal basis.

In this exercise we study the symmetries of the cube \( C = [-1,1]^3 \) and the dodecahedron \( D \), as defined in the book. We write \( H = S(C) \cap S(D) \).

We use the notations in the book for the vertices \( v_0, v_1 \in D \), the face \( F \) of \( D \) and the rotation \( \rho_F \) that permutes the vertices of \( F \).

(a) Give the matrices for the following transformations and say whether the transformations lie in \( S(C), S(D) \) and/or \( H \).

(i) \( \rho_{(e_3, \frac{\pi}{2})} \)
(ii) \( \rho_{(e_1, \frac{\pi}{2})} \)
(iii) \( \rho_{(e_3, \frac{\pi}{2})}\rho_{(e_1, \frac{\pi}{2})} \)
(iv) \( \rho_{(e_1+e_2, \pi)} \)
(v) \( \rho_{(v_0, \frac{\pi}{4})} \)
(vi) \( \rho_F. \)
(b) Calculate the matrices for the following products and then show what rotations they are (i.e., write them in the form $\rho(x, \theta)$ for some $x$ and $\theta$). What is the order of these transformations (i.e., what is the smallest positive $k$ such that their $k$-th power is the identity)?

(i) $\rho(v_0, \frac{2\pi}{3}) \rho_F$

(ii) $\rho(v_0, \frac{4\pi}{3}) \rho_F$

(iii) $\rho(e_3, \frac{\pi}{2}) \rho(e_1, \frac{\pi}{2})$