

Math 432-532 Syllabus, Fall 2009

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Office	ES 136A
Hours	MW 1:40–2:10 & 3:45–4:15; F 1:40-2:35; and by arrangement
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	Please include Math 432-532 in the subject line.
Texts	<i>Euclidean and non-Euclidean geometry</i> , Patrick Ryan <i>Hyperbolic geometry</i> , James Anderson
My home page	http://math.albany.edu/~mark
Course home page	http://math.albany.edu/~mark/classes/432-532/

Euclidean geometry is the geometry of the visible plane and the properties of its lines, circles, polygons, etc. The Greeks, as codified by Euclid, set up axioms that led to formal derivations of the visible properties of these shapes.

Today, when we take calculus, we derive the same properties from calculus and analytic geometry. Instead of using Euclid's axioms, we use the axioms of set theory and of the real line, obtaining much more information about these visible shapes than was available from Euclid's axioms. For instance, we can derive the very powerful laws of cosines and of sines, taught today in high school.

So one way of thinking about analytic geometry is that it is a realization of Euclid's axioms: a system that satisfies Euclid's axioms and much more. A realization that is fundamental to the foundations of modern mathematics.

From the very beginning in Greek times, there was controversy about Euclid's parallel postulate: that given a line ℓ and a point P not on it there is a unique line m through P parallel to ℓ . From the standpoint of the visible or naive geometry of the plane it seems clearly true, and many thought it could be derived from Euclid's other four axioms.

Of course, in analytic geometry, the parallel postulate is an easy theorem: a line m is parallel to ℓ if and only if they have the same slope. And the point-slope formula shows that there is a unique line m through P with a prescribed slope.

Let us now segue to the study of non-Euclidean geometry. In the 19th century, Bolyai, Lobachevsky and others showed that there is a viable axiomatic system of geometry based on the first four of Euclid's axioms (those other than the parallel postulate) together with the assumption that given a line ℓ and a point P not on it, there is more than one line through P parallel to ℓ .

Poincaré was able to provide a realization for the non-Euclidean axioms using analytic tools analogous to those used in “analytic geometry”. The tools in question are called Riemannian geometry. In fact, Poincaré’s realization of non-Euclidean geometry comes from putting a nonstandard Riemannian structure on the plane! The resulting geometric structure is called hyperbolic space.

Another geometry of obvious importance to us is spherical geometry: the geometry of great circles on the sphere. The key property of great circles is that they are curves that realize shortest distances between points (as calculated via the arc length formula in calculus). Because they are distance-minimizing curves (or geodesics, in the language of Riemannian geometry) they generate the geometry of importance for things like plotting travel routes by airplanes. Given three cities, the spherical “triangle” that joins them is obtained from the great circles joining each pair of points.

Spherical geometry can be studied using the standard geometric structure of 3-dimensional space, i.e., the standard tools in the calculus and linear algebra of several variables. No new structure need be introduced.

Note that spherical geometry is not planar: it is realized on the sphere and not the plane.

The three geometries discussed above will be derived and studied using analytic tools. The basic properties of triangles in each context will be studied. An interesting fact is that while the angles in a Euclidean triangle always add up to π radians, the angles in a spherical triangle can add up to any number in the open interval $(\pi, 3\pi)$. In hyperbolic space, the angle sum for a triangle is always less than π .

This fact about angle sums in the three geometries is related to a concept in Riemannian geometry known as curvature. The curvature of Euclidean space is 0 at each point. The curvature of the sphere is 1 at each point. The curvature of hyperbolic space is -1 at each point. We will not discuss curvature in this class, but it is a useful concept.

We will also discuss the analogues of the laws of cosines and sines that hold in the spherical and hyperbolic geometries.

A very important concept that will be developed in all three geometries is the group of isometries, or distance-preserving transformations, of each of these spaces. The isometries of the Euclidean plane are studied in greater detail in Math 331 and Math 531.

Note that there are two textbooks. Ryan will be used for Euclidean and spherical geometry, and Anderson for hyperbolic geometry. Ryan’s book does contain a derivation of hyperbolic geometry, but it is less intuitive, in my opinion, than the Poincaré model developed in Anderson’s book. The model in Ryan’s book is based on Einsteinian metrics (rather than Riemannian ones), and has independent interest.

There will be three in-class exams, one for each of the three geometries, the dates of which will be announced in class one week prior to each exam. There will also be graded in-class projects or quizzes every class period. The projects and quizzes will be based on having studied the preceding lecture(s), and also any homework problems assigned. Homework will not be graded.

There is no final exam.

Each exam counts for 27% of your grade, and the in-class work counts for 19%.

Please do ask lots of questions. Your questions are a very good indicator of what you understand. My goal here is to teach you, not to penalize you. The test for all of us is how you do on the exams, projects and quizzes. So please make use of the class and office hours to get my help. I am happy to give it. Remember, if you have a question, there will be at least 5 others in the class with the same question. I hope one of you will ask it, because that is how we learn.

Most people learn mathematics more quickly and thoroughly if they discuss it. Verbalizing a question is often the most important step in solving it. So please make frequent use of office hours. It is also very useful to get to know your fellow students and form study groups.

The ultimate test is being able to solve problems. Keep your curiosity alive and follow it where it leads.

Academic policies.

Attendance. Your in-class performance is key to your success in this course. Attendance, itself, is not graded. Instead, graded in-class activities and assignments constitute an important part of the course grade. Keeping a passing average on these is not possible without consistent attendance. Missing class (or part of a class) means the student earns an automatic “0” credit for the activities or assignments missed. See below for policies related to extreme emergencies and extended illnesses.

Make-up policy. Since there will be occasions in your life when missing a class meeting is simply unavoidable, this course has two NO-FAULT safety valves.

Safety valve 1. You may drop 3 in-class assignment grades. So, if you must miss class for any reason, it will be possible (up to 3 times) to drop the zero you would automatically receive for missing the assignment. You can use this safety valve for both legitimate and non-legitimate absences—you do not need to warn me or bring documentation or offer explanations. This safety valve is for you to manage. Be careful not to waste your drops on frivolous things early in the semester, since you may need them if you catch a cold or need to leave town for a day, later in the semester. If you do not use your safety valve for missed classes, you will be able to use your safety valve to

improve your grade, by dropping any low scores you make even when you DO attend.

Plan carefully for classes that you know you will need to miss. It is your responsibility to manage your absences by using your safety valves. If you need to be out of class for any reason, make sure you have conserved your “droppable” grades to cover the class you need to miss.

Safety valve 2. If you become seriously ill during the semester, or become derailed by unforeseeable life problems, and have to miss so many assignments that it will ruin your grade, you and I will schedule a special meeting in order to make arrangements for you to take the appropriate administrative steps. Don't wait until too late to see me when you get in trouble.

Cell phones. Please mute your cell during class. Texting during class is not permitted.

Academic honesty policy. Students are on their honor to be ethical and honest in carrying out all the assignments and requirements of this course. Any violations of this code, such as cheating, copying, plagiarism, or misrepresentation of one's own work, will meet with the appropriate penalties and discipline as outlined in UAlbany regulations. If you have an unclear picture of what constitutes plagiarism, or the limits of acceptable group collaboration, please ask the instructor for clarification.

It's also the responsibility of every student to report any observed instances of cheating and plagiarism.

Disciplinary actions for such offenses are severe, and include loss of course credit, suspension, and expulsion from the university.

Special needs. Students who have special needs due to learning or other disabilities will be accommodated, and should inform the instructor at the beginning of the semester. Students who request accommodation will be asked to provide appropriate documentation, which may be obtained through the student services office.