1. Let $V$ be a finite dimensional vector space over the complex numbers and let $T \in L(V, V)$ such that every $T$-invariant subspace has a $T$-invariant complement. Show that $T$ is diagonalizable.

Do not use Jordan canonical form in your solution.

2. Let $A = J(c, n_1) \oplus \cdots \oplus J(c, n_k)$ with $n_1 \geq \cdots \geq n_k$.
   a) What are $\min_A(x)$ and $\ch_A(x)$?
   b) What is $\dim \ker (A - cI)$?
   c) What is the rational canonical form of $A$?

3. Let $T \in L(V, V)$, and suppose that $\min_T(x) = p^4$ and $\ch_T(x) = p^8$ where $p \in F[x]$ is irreducible. List all possible rational canonical forms for $T$.

4. Find a pair of complex matrices $A$ and $B$ such that
   a) $\ch_A(x) = \ch_B(x)$.
   b) $\min_A(x) = \min_B(x)$.
   c) For each eigenvalue $c$ of $A$ (hence also of $B$), the eigenspaces of $A, c$ and of $B, c$ have the same dimension.

5. Let
   $$A = \begin{bmatrix}
   2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
   1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
   -4 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\
   2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
   -7 & 2 & 0 & 0 & 2 & 0 & 0 & 0 \\
   9 & 0 & -2 & 0 & 1 & 2 & 0 & 0 \\
   -34 & 7 & 1 & -2 & -1 & 1 & 2 & 0 \\
   145 & -17 & -16 & 3 & 9 & -2 & 0 & 3 
   \end{bmatrix}.$$

   a) What is the characteristic polynomial of $A$?
   b) For each eigenvalue $c$, find $\dim \ker (A - cI)^k$ for each $k$ less than or equal to the multiplicity of $c$ in $\ch_A(x)$.
   c) What is the minimal polynomial of $A$?
   d) What is the Jordan canonical form of $A$? (Write it as a formal block sum of formal Jordan blocks.)
   e) What is the rational canonical form of $A$? (Write it as a formal block sum of formal companion matrices.)
   f) Find an element whose $A$-annihilator is $(x - 2)^2$. 