

1. A committee consists of 8 men and 7 women. Pick 5 at random.
 - a) What is the probability you get more men than women?
 - b) What is the probability you get more men than women if there are at least two women?
2. Find
 - a) the expected value
 - b) the variancefor the number of suits in a 7-card poker hand.
3. Find
 - a) the expected value
 - b) the variancefor the number of aces in a 5-card poker hand.
4. Urn A contains 7 red balls and three green balls. Urn B contains 5 red balls and 5 green balls. Pick an urn at random and draw two balls without replacement. Suppose they both come up red.
 - a) What is the probability they came from Urn A?
 - b) If a third ball is drawn from the same urn (without replacement), what is the probability it is red?
5. Flip three coins. Then draw letters with replacement from a word, to be determined below, until you get a vowel.

If you get no heads, draw from APPLE.
If you get one head, draw from PEAR.
If you get two heads, draw from PLUM.
If you get three heads, draw from KUMQUAT.

How many letters do you expect to draw?

6. Suppose that

$$f(z) = \frac{4}{(3-z)^2}$$

is the generating function for a random variable X . Find

- a) $P(X > 0)$
- b) $P(X > 1)$
- c) $E(X)$
- d) $\text{Var}(X)$

7. Consider the transition matrix

$$\begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

- a) Find R_i for each state $i = 1, \dots, 8$.
- b) Classify the recurrent and the transient states.
- c) If you start in E_3 , what is the probability you will return there?
- d) If you start in E_2 , how many times do you expect to visit E_3 ?
- e) For each transient state, compute the expected number of steps needed to reach a recurrent state.