

1. How many ways can you arrange the letters in
TUSCALOOSA MISSISSIPPI
with no two adjacent vowels?
2. A certain kind of poker has 6 cards. What is the probability you get a full house in it (i.e., three cards in one denomination and at least two cards in another).
3. A certain country has a careless mint. One coin in every 501 manufactured has heads on both sides. The remaining coins have heads on one side, tails on the other, as usual. A randomly selected coin made at the mint is tossed 6 times, and comes up heads each time.
 - a) What is the probability the coin is two-headed?
 - b) What is the probability it will come up heads again if it is tossed a 7-th time?Note: the above two probabilities are conditional on the coin having come up heads 6 times in a row.
4. You have 5 red balls, 5 green balls, 5 blue balls, and 5 white balls. Pick 6 balls at random without replacement. What is
 - a) the expected value
 - b) the variancefor the number of colors you get?
5. Flip four coins. Then draw from a deck with replacement until the number of spades you've drawn equals the number of heads from your coin toss. How many cards do you expect to draw?
6. An urn contains 6 red balls and 5 green balls. Draw 5 times without replacement. What is the probability you get at least 2 red balls if you get at least one green one?
7. Suppose that

$$f(z) = \frac{4}{(3-z)^2}$$

- is the generating function for a random variable X . Find
- a) $P(X > 0)$
 - b) $E(X)$
 - c) $\text{Var}(X)$

Final Exam

8. Peter goes to the casino with \$500. He will bet \$1 at a time, with a probability of .45 of winning. He will leave if he ever gets \$50 ahead, or, alternatively, if he goes broke.
- What is the probability Peter wins?
 - How many bets can he expect to make?

9. Consider the transition matrix

$$\begin{bmatrix} 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

- Find R_i for each state $i = 1, \dots, 7$.
- Classify the recurrent and the transient states.
- Compute h_{62} .
- For each transient state, compute the expected number of steps needed to reach a recurrent state.