

1. Fred goes to the casino with \$100. He decides he's going to keep betting, \$1 at a time for even money, on a game where he has a probability of .47 of winning, until his money runs out. How long can he expect to play?
2. Peter bets \$1 at a time for even money on a game he has a probability of .41 of winning. If you give him unlimited credit, what is the probability he'll ever get \$10 ahead?
3. Consider the transition matrix

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

- a) Find R_i for each state $i = 1, \dots, 8$.
- b) Classify the recurrent and the transient states.
- c) Compute h_{43} .
- d) Compute h_{45} .
- e) Compute v_{12} .
- f) Compute r_{13} and r_{23} .
- g) For each transient state, compute the expected number of steps needed to reach a recurrent state.