Math 367 **Exam 3** **Fall '99**

1. Suppose that
$$f(z) = \frac{1}{(2-z)^2}$$
is the generating function for a random variable X. Find
a) $\mathrm{P}(X > 0)$ b) $\mathrm{P}(X > 1)$ c) $\mathrm{E}(X)$ d) $\mathrm{Var}(X)$

2. George goes to the casino with \$200. He decides to keep betting, \$1 at a time for even money, on a game where he has a probability of .45 of winning, until his money runs out. How long can he expect to play?

3. Bob goes to the casino with \$300. He will bet \$1 at a time, with a probability of .47 of winning. He will leave if he ever gets \$50 ahead, or, alternatively, if he goes broke.
a) What is the probability Bob gets the \$50?
b) How many bets can he expect to make?

4. Albert bets \$1 at a time for even money on a game he has a probability of .43 of winning. If you give him unlimited credit, what is the probability he'll ever get \$10 ahead?

5. Consider the transition matrix
$$\begin{bmatrix} \frac34 & \frac14 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac14 & \frac14 & \frac14 & 0 & 0 & 0 & 0 & \frac14 \\ 0 & 0 & \frac12 & 0 & 0 & \frac14 & 0 & \frac14 \\ 0 & 0 & 0 & \frac14 & 0 & 0 & \frac34 & 0 \\ 0 & 0 & \frac14 & \frac14 & \frac14 & 0 & 0 & \frac14 \\ 0 & 0 & 0 & 0 & 0 & \frac12 & 0 & \frac12 \\ 0 & 0 & 0 & \frac12 & 0 & 0 & \frac12 & 0 \\ 0 & 0 & \frac12 & 0 & 0 & 0 & 0 & \frac12 \end{bmatrix}$$
a) Find R_i for each state $i = 1, \ldots, 8$.
b) Classify the recurrent and the transient states.
c) If you start in E_2, what is the probability you will return there?
d) If you start in E_1, how many times do you expect to visit E_2?
e) If you start in E_1, how long do you expect it to take you to reach E_2?
f) For each transient state, compute the expected number of steps needed to reach a recurrent state.