Math 367 **Exam 3** **Spring '04**

1. Suppose that
$$f(z) = \frac{6}{(7-z)}$$
is the generating function for a random variable X. Find
a) $\mathrm{P}(X > 2)$ b) $\mathrm{E}(X)$ c) $\mathrm{Var}(X)$

2. Consider the transition matrix
$$\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
a) Find R_i for each state $i = 1, \ldots, 8$.
b) Classify the recurrent and the transient states.
c) If you start in E_3, what is the probability you will return there?
d) If you start in E_6, how many times do you expect to visit E_3?
e) If you start in E_3, how long do you expect it to take you to reach E_1?
f) From each transient state, how long do you expect it take you to reach a recurrent state?

3. You roll a die until you get the sequence 1,1,2,2. What is the transition matrix for the Markov system describing this game?

4. Find a fixed probability vector for the transition matrix
$$\begin{bmatrix} \frac{1}{4} & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$