Math 367 **Exam 3** **Fall '02**

1. Suppose that
$$f(z) = \frac{4}{(5-z)}$$
is the generating function for a random variable X. Find
a) $\mathrm{P}(X > 1)$ b) $\mathrm{E}(X)$ c) $\mathrm{Var}(X)$

2. Bob goes to the casino with \$100. He will bet \$1 at a time, with a probability of .43 of winning. He will leave if he ever gets \$25 ahead, or, alternatively, if he goes broke.
a) What is the probability Bob gets the \$25?
b) How many bets can he expect to make?

3. Albert bets \$1 at a time for even money on a game he has a probability of .48 of winning. If you give him unlimited credit, what is the probability he'll ever get \$12 ahead?

4. Consider the transition matrix
$$\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$
a) Find R_i for each state $i = 1, \ldots, 8$.
b) Classify the recurrent and the transient states.
c) If you start in E_2, what is the probability you will return there?
d) If you start in E_3, how many times do you expect to visit E_2?
e) If you start in E_7, how long do you expect it to take you to reach E_4?
f) If you start in E_2, how long do you expect it take you to reach a recurrent state?

5. You roll a die until you get the sequence 1,2,1,2. What is the transition matrix for the Markov system describing this game?