

1. A certain country has a careless mint. One out of every 801 coins comes out with two heads. Pick a coin at random and flip it three times.
  - a) What is the probability all three tosses are heads?
  - b) What is the conditional probability the coin was two-headed, given that all three tosses were heads?
2. George decides to play roulette, betting on a single number each time, until he wins. Since a roulette wheel has 38 slots, the probability of winning on a given play is  $\frac{1}{38}$ .
  - a) What is the probability he plays at least 5 times?
  - b) How many times should he expect to play?
  - c) What is the variance for the number of plays?
3. Deal 7 cards from a well-shuffled deck.
  - a) What is the probability at least five of the cards are spades?
  - b) What is the probability at least five of the cards are in the same suit?
  - c) How many spades do you expect to get?
  - d) What is the variance for the number of spades in the hand?
4. Consider the density function

$$f(x, y) = \begin{cases} \frac{15}{2}xy^2 & \text{for } 0 \leq x \leq 1, -x \leq y \leq x. \\ 0 & \text{otherwise.} \end{cases}$$

Compute the following:

- a)  $f_1(x)$
- b)  $E(X)$
- c)  $V(X)$
- d)  $E(Y|X)$
- e)  $V(Y|X)$
- f)  $E(Y)$
- g)  $E(Y^2)$
- h)  $V(Y)$
- i)  $E(XY)$
- j)  $\text{Cov}(X, Y)$
- k)  $\rho$
- l)  $E(2X + 3Y)$
- m)  $V(2X + 3Y)$ .

- n) If  $0 \leq y \leq 1$ , what is  $f_2(y)$ ?
  - o) Are  $X$  and  $Y$  independent? Give a reason.
  - p) What is  $P(Y \geq -\frac{1}{4} \mid X \leq \frac{1}{2})$ ?
  - q) What is  $P(Y \geq -\frac{1}{4} \mid X = \frac{1}{2})$ ?
5. A large jar is filled with coins. 40% of the coins are quarters, 30% are dimes, 20% are nickels and 10% are pennies. Select 10 coins at random.
- a) What is the probability five of the coins are quarters, two are dimes, two are nickels and one is a penny?
  - b) What are the expected value and variance for the amount of money drawn?