

1. Consider the density function

$$f(x, y) = \begin{cases} 15x^2y & \text{for } 0 \leq x \leq 1, x \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a) What is the marginal density function  $f_2(y)$ ?
- b) What is  $P(Y \geq \frac{1}{2})$ ?
- c) What is  $P(X \leq \frac{1}{4} \mid Y = \frac{1}{2})$ ?

2. Consider the density function

$$f(x, y) = \begin{cases} \frac{3}{2}(x + y) & \text{for } 0 \leq x \leq 1, -x \leq y \leq x \\ 0 & \text{otherwise.} \end{cases}$$

- a) What is the marginal density function  $f_1(x)$ ?
- b) What is  $P(X \leq \frac{3}{4}, Y \leq \frac{1}{2})$ ?
- c) What is  $P(X \leq \frac{3}{4})$ ?
- d) What is  $P(Y \leq \frac{1}{2} \mid X \leq \frac{3}{4})$ ?

3. Let

$$f(y) = \begin{cases} ky^3 & \text{for } 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a) For what value of  $k$  is this a density function?
- b) For this value of  $k$ , what are the expected value and variance of  $Y$ ?