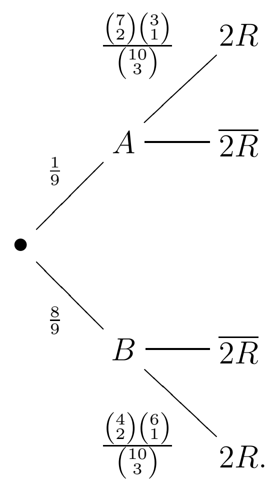
# Math 362 Exam 1 solutions Fall '09

1. Urn A contains 7 red balls and 3 green balls. Urn B contains 4 red balls and 6 green balls. An experiment is conducted as follows:

   > Roll a pair of dice. If the sum is 5 draw three balls from Urn A without replacement. Otherwise, draw three balls from Urn B without replacement.

   a) What is the probability that exactly two of the balls are red?
   b) What is the conditional probability you drew from Urn A given that exactly two of the balls were red?

   SOLUTION: Let A be the event that the balls were drawn from Urn A and let B be the complementary event. Let "2R" be the event that exactly two of the balls are red. We have the following tree diagram:




   Here, $\frac{1}{9}$ comes from the fact that exactly four of the 36 outcomes for rolling a pair of dice have a sum of 5: $(1,4)$, $(2,3)$, $(3,2)$ and $(4,1)$. The conditional probabilities of getting exactly two red balls given that the drawing comes from the specified urn come from the hypergeometric distribution.

   The probability that exactly two balls are red is given by the sum of the probabilities of the paths ending in 2R:

$$\frac{1}{9} \cdot \frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}} + \frac{8}{9} \cdot \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} = .325.$$

The conditional probability you drew from Urn A given that exactly two balls are red is

$$\frac{\frac{1}{9}\cdot\frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}}}{\frac{1}{9}\cdot\frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}}+\frac{8}{9}\cdot\frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}}=\frac{\binom{7}{2}\binom{3}{1}}{\binom{7}{2}\binom{3}{1}+8\binom{4}{2}\binom{6}{1}}=\frac{7}{7+32}=\frac{7}{39}\approx .1795.$$

2. A bomber pilot (Catch 22) has to make one bombing run per week until he gets shot down. On each bombing run he has a probability of .95 of not being shot down.

   SOLUTION: This is a geometric distribution, where "success" means the string of bombing runs ceases, i.e., that he gets shot down. So $p = .05$ and $q = .95$.

   a) What is the probability he makes at least four bombing runs?
      SOLUTION: This is 1 minus the probability of at most three bombing runs, or $1-(p+qp+q^2p)$. Alternatively, this can be calculated as

$$q^3p+q^4p+q^5p+\cdots=q^3p(1+q+q^2+\cdots)=q^3p\cdot\frac{1}{1-q}=q^3p\cdot\frac{1}{p}$$
$$=q^3\approx .8574.$$

   b) How many runs can he expect to make?
      SOLUTION: The expected value of the geometric distribution is $\frac{1}{p}=20$.

   c) What is the variance for the number of bombing runs he'll make?
      SOLUTION: The variance of the geometric distribution is $\frac{q}{p^2}$.

3. What is the probability of getting a full house in seven card poker, i.e., no more than three of any kind, three of at least one kind, and at least two of another kind.

   SOLUTION:

$$\frac{\binom{13}{2}\binom{4}{3}\binom{4}{3}\binom{11}{1}\binom{4}{1}+\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{2}\binom{4}{2}+\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}\binom{11}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{7}}.$$

4. There are 9 men and 11 women in a group. 7 are chosen at random.

   a) What is the probability you get more women than men?
      SOLUTION: This is hypergeometric:

$$\frac{\binom{9}{3}\binom{11}{4}+\binom{9}{2}\binom{11}{5}+\binom{9}{1}\binom{11}{6}+\binom{11}{7}}{\binom{20}{7}}.$$

b) How many women do you expect there to be?
SOLUTION: The expected value is $np$, where $n = 7$, the "sample size", and $p = \frac{11}{20}$, the percentage of women in the group.

c) What is the variance for this question?
SOLUTION: The variance is $npq \cdot \frac{20-7}{20-1}$ with $n$ and $p$ as above and with $q = 1 - p$.

5. There are 300 misprints in a 100 page document. Suppose the number of misprints on a given page follows a Poisson distribution. What is the probability there at least two misprints on page 57?

SOLUTION: The expected value is 3 misprints per page, i.e., $\lambda = 3$. So $P(Y = k) = \frac{3^k}{k!}e^{-3}$. The probability of at least two misprints is $1 - (e^{-3} + 3e^{-3})$.

6. Joe repeatedly plays a game where he has a 1/3 chance of winning each time. Suppose he plays it 5 times.

SOLUTION: This is a binomial distribution with $n = 5$ and $p = \frac{1}{3}$.

a) What is the probability he loses more often than he wins?
SOLUTION: $(2/3)^5 + \binom{5}{1}(1/3)(2/3)^4 + \binom{5}{2}(1/3)^2(2/3)^3$.

b) What is the expected number of wins?
SOLUTION: The expected value is $np = \frac{5}{3}$.

c) What is the variance for the number of wins?
SOLUTION: The variance is $npq = \frac{10}{9}$. Here, $q = 1 - p = \frac{2}{3}$.