The following are wallpaper patterns. On each one, indicate the following with colored ink:

- Shortest translations, $\tau_v$ and $\tau_w$, in two different directions, that preserve the pattern and form the boundary of a fundamental region $R$ for $T$.
- All $n$-centers for each possible $n$.
- All lines of symmetry.
- If there are glide reflections but no reflections, give the axes for the glide reflections.
- A fundamental region, $R$, for $T$. If $W$ is a $W_3$-group that contains lines of symmetry, base it at a 3-center on a line of symmetry. Otherwise base it at an $n$-center for the largest possible $n$.
- A fundamental region, $S$ for $W$.

1. a) How many $T$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? SOLUTION: No centers.
   b) Which wallpaper group is $W$? SOLUTION: $W_1$. There are no symmetries other than translations.
2. Wallpaper patterns

a) How many $T$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? SOLUTION: No centers.
b) Which wallpaper group is $W$? SOLUTION: $W_3^6 = K$, the Klein bottle group. There are glide reflections but no reflections.

3. Wallpaper patterns

a) How many $T$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? SOLUTION: Two $T$-orbits of 4-centers, both $D_8$, and two $T$-orbits of 2-centers, both $D_4$.
b) Which wallpaper group is $W$? SOLUTION: $W_4^1$. 

4. a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy?  

Solution: Two $\mathcal{T}$-orbits of 4-centers, both $C_4$, and two $\mathcal{T}$-orbits of 2-centers, both $C_2$.  

b) Which wallpaper group is $\mathcal{W}$?  

Solution: $\mathcal{W}_4$.  

5. a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy?  

Solution: Two $\mathcal{T}$-orbits of 4-centers, both $C_4$, and two $\mathcal{T}$-orbits of 2-centers, both $D_4$.  

b) Which wallpaper group is $\mathcal{W}$?  

Solution: $\mathcal{W}_4^2$.  

6. a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? SOLUTION: Four $\mathcal{T}$-orbits of 2-centers, two $D_4$ and two $C_2$.

b) Which wallpaper group is $W$? SOLUTION: $W_2^1$.

7. a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? SOLUTION: Four $\mathcal{T}$-orbits of 2-centers, all $D_4$.

b) Which wallpaper group is $W$? SOLUTION: $W_2^2$.

8. a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? SOLUTION: Four $\mathcal{T}$-orbits of 2-centers, all $C_2$.

b) Which wallpaper group is $W$? SOLUTION: $W_2$. 
Wallpaper patterns

9.

a) How many $T$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? Solution: Four $T$-orbits of 2-centers, all $C_2$.

b) Which wallpaper group is $W$? Solution: $W_3^2$: there are glide reflections but no reflections.

10.

a) How many $T$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? Solution: Four $T$-orbits of 2-centers, all $C_2$.

b) Which wallpaper group is $W$? $W_2^3$: there are reflections, but not through any points of symmetry.
11. a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? Solution: Three $\mathcal{T}$-orbits of 3-centers, all $D_6$.

b) Which wallpaper group is $\mathcal{W}$? Solution: $\mathcal{W}_3^1$.

12. a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? Solution: Three $\mathcal{T}$-orbits of 3-centers, one $D_6$ and the others $C_3$.

b) Which wallpaper group is $\mathcal{W}$? Solution: $\mathcal{W}_3^2$. 
13. a) How many \( T \)-orbits are there of \( n \)-centers for each possible \( n \) and what is their isotropy? SOLUTION: Three \( T \)-orbits of 3-centers, all \( C_3 \).

b) Which wallpaper group is \( W \)? SOLUTION: \( W_3 \).

14. a) How many \( T \)-orbits are there of \( n \)-centers for each possible \( n \) and what is their isotropy? SOLUTION: One \( T \)-orbit of 6-centers, \( C_6 \). Two \( T \)-orbits of 3-centers, both \( C_3 \). Three \( T \)-orbits of 2-centers, all \( C_2 \).

b) Which wallpaper group is \( W \)? SOLUTION: \( W_6 \).
15.

a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? **SOLUTION:** One $\mathcal{T}$-orbit of 6-centers, $D_{12}$. Two $\mathcal{T}$-orbits of 3-centers, both $D_6$. Three $\mathcal{T}$-orbits of 2-centers, all $D_4$.

b) Which wallpaper group is $W$? **SOLUTION:** $W_6^1$.

16.

a) How many $\mathcal{T}$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? **SOLUTION:** no centers.

b) Which wallpaper group is $W$? **SOLUTION:** $W_1^1$: rhombic fundamental region for $\mathcal{T}$ with a reflection across one of the diagonals.
17. a) How many $T$-orbits are there of $n$-centers for each possible $n$ and what is their isotropy? SOLUTION: no centers.
b) Which wallpaper group is $W$? $W^2_1$: rectangular fundamental region for $T$ with reflections parallel to one pair of opposite edges.