

1. The following are frieze patterns. On each one, indicate the following with colored ink:

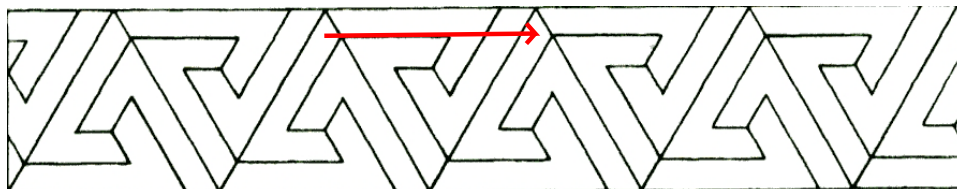
- The shortest translation,  $\tau$ , that preserves the pattern.
- All points of symmetry (i.e., all points  $P$  for which  $\rho_{P,\pi}$  is in the frieze group,  $\mathcal{F}$ ).
- All vertical lines of symmetry (i.e., all vertical lines  $n$  such that  $\sigma_n \in \mathcal{F}$ ).

(a) WOIOM OIOWOIOM

SOLUTION: The shortest translation takes each W to the next W. There is a point of symmetry in the middle of each I and vertical lines of symmetry through the middle of each W and the middle of each M.

- (i) Is there a horizontal line of symmetry? SOLUTION: No.
- (ii) Is there a glide reflection whose square is  $\tau$ ? SOLUTION: Yes.
- (iii) Which of the seven listed groups is  $\mathcal{F}$ ? SOLUTION:  $\mathcal{F}_2^2$ .

(b)



SOLUTION:  $\tau$  is represented by the arrow. There are no vertical lines of symmetry and no points of symmetry.

- (i) Is there a horizontal line of symmetry? SOLUTION: No.
- (ii) Is there a glide reflection whose square is  $\tau$ ? SOLUTION: Yes.
- (iii) Which of the seven listed groups is  $\mathcal{F}$ ? SOLUTION:  $\mathcal{F}_1^3$ .

(c)



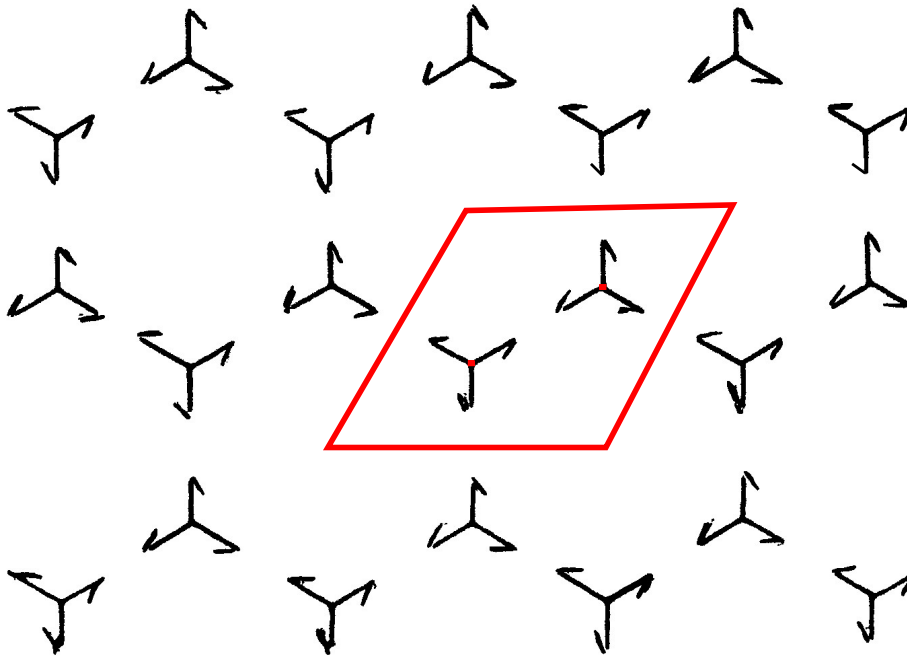
SOLUTION:  $\tau$  is represented by the arrow. There are no vertical lines of symmetry. The red dots denote points of symmetry.

- (i) Is there a horizontal line of symmetry? SOLUTION: No.
- (ii) Is there a glide reflection whose square is  $\tau$ ? SOLUTION: No.
- (iii) Which of the seven listed groups is  $\mathcal{F}$ ? SOLUTION:  $\mathcal{F}_2$ .

2. The following are wallpaper patterns. On each one, indicate the following with colored ink:

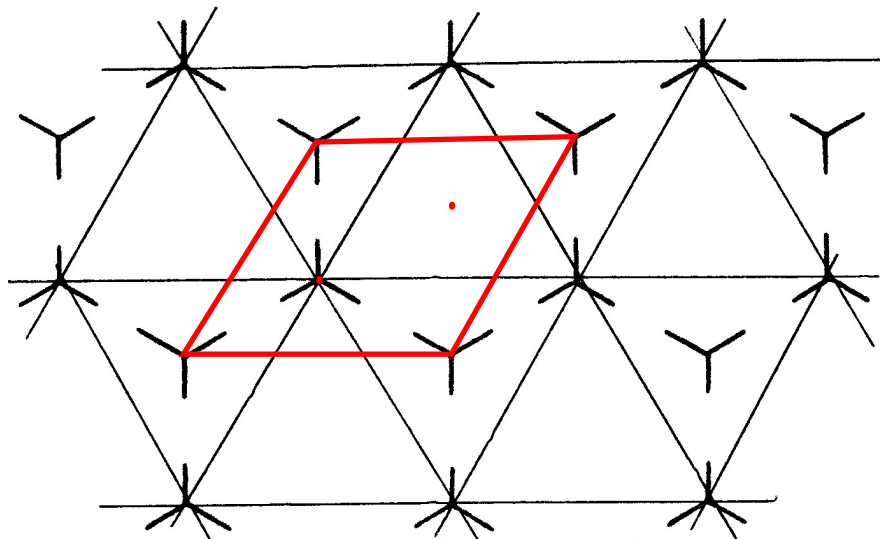
- A rhombic fundamental region for the action of the translation subgroup  $\mathcal{T}$  of the wallpaper group  $\mathcal{W}$  for this pattern. (The sides of the rhombus are shortest translations in  $\mathcal{W}$ .) If there are 6-centers for this pattern, use them for vertices of this region. Otherwise place the vertices at 3-centers.
- The 3-centers that lie in the fundamental region.

(a)



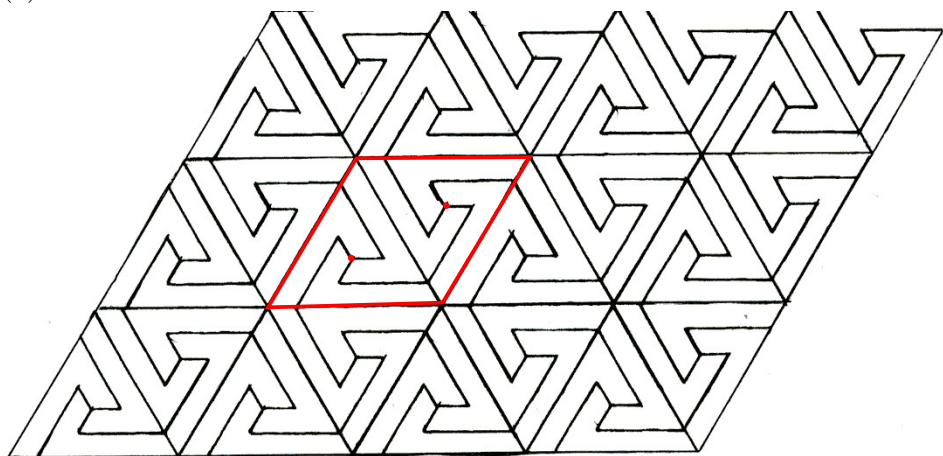
- (i) Which of the diagonals (long, short or both) of the fundamental region are lines of symmetry for the pattern? SOLUTION: Neither.
- (ii) Which of the listed wallpaper groups is  $\mathcal{W}$ ? SOLUTION:  $\mathcal{W}_6$

(b)



- (i) Which of the diagonals (long, short or both) of the fundamental region are lines of symmetry for the pattern? SOLUTION: The long diagonal.
- (ii) Which of the listed wallpaper groups is  $\mathcal{W}$ ? SOLUTION:  $\mathcal{W}_3^1$

(c)



- (i) Which of the diagonals (long, short or both) of the fundamental region are lines of symmetry for the pattern? SOLUTION: The short diagonal.
- (ii) Which of the listed wallpaper groups is  $\mathcal{W}$ ? SOLUTION:  $\mathcal{W}_3^2$