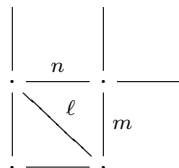


1. Compute $\rho_{[1], \frac{\pi}{2}} \rho_{[0], \frac{\pi}{2}}$ explicitly.

SOLUTION: Write ℓ for the line $y = -x + 1$, m for the line $x = 1$ and n for the line $y = 1$:



Then $\rho_{[1], \frac{\pi}{2}} = \sigma_n \sigma_\ell$ and $\rho_{[0], \frac{\pi}{2}} = \sigma_\ell \sigma_m$, so

$$\rho_{[1], \frac{\pi}{2}} \rho_{[0], \frac{\pi}{2}} = \sigma_n \sigma_\ell \sigma_\ell \sigma_m = \sigma_n \sigma_m.$$

Since $n \cap m = [1]$ and the directed angle from m to n is $\frac{\pi}{2}$, $\sigma_n \sigma_m = \rho_{[1], \pi}$.

2. Compute $\tau_{O, [2]} \rho_{O, \frac{\pi}{3}}$ explicitly.

SOLUTION: This one is harder to draw electronically. Let ℓ be the line $y = 0$ and m the line $y = 1$. Then $\ell \parallel m$ and the directed distance from ℓ to m is $[1]$, so $\tau_{O, [2]} = \sigma_m \sigma_\ell$. Let n be the line $y = -\frac{1}{\sqrt{3}}x$. Then the directed angle from n to ℓ is $\frac{\pi}{6}$, so $\rho_{O, \frac{\pi}{3}} = \sigma_\ell \sigma_n$. Thus,

$$\tau_{O, [2]} \rho_{O, \frac{\pi}{3}} = \sigma_m \sigma_\ell \sigma_\ell \sigma_n = \sigma_m \sigma_n.$$

Because ℓ and m are parallel, the directed angle from n to m is $\frac{\pi}{6}$, so $\tau_{O, [2]} \rho_{O, \frac{\pi}{3}} = \rho_{P, \frac{\pi}{3}}$, where $P = m \cap n$. But we can easily solve for P : from m , we have $y = 1$. From n , then, $1 = -\frac{1}{\sqrt{3}}x$, so $x = -\sqrt{3}$, hence $P = [-\sqrt{3}]$.

3. Let ℓ be the line $y = 2x + 5$, m the line $y = 2x + 1$, and n the line $y = -\frac{1}{2}x$. Calculate the following explicitly:

a) $\sigma_n\sigma_\ell$

SOLUTION: $\ell \perp n$, so $\sigma_n\sigma_\ell = \rho_{P,\pi}$, where $P = \ell \cap n$. To find P , set $2x + 5 = -\frac{1}{2}x$. We get $x = \frac{2}{5}(-5) = -2$. So $y = -\frac{1}{2}(-2) = 1$, hence $P = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

b) $\sigma_n\sigma_m$

SOLUTION: $m \perp n$, so $\sigma_n\sigma_m = \rho_{Q,\pi}$, where $Q = m \cap n$. To find Q , set $2x + 1 = -\frac{1}{2}x$. We get $x = \frac{2}{5}(-1) = -\frac{2}{5}$. So $y = -\frac{1}{2}(-\frac{2}{5}) = \frac{1}{5}$, hence $Q = \begin{bmatrix} -\frac{2}{5} \\ \frac{1}{5} \end{bmatrix}$.

c) $\sigma_m\sigma_\ell$.

SOLUTION: $m \parallel \ell$, so $\sigma_m\sigma_\ell$ is the translation by twice the directed distance from ℓ to m . Since n is perpendicular to ℓ and m , the directed distance from ℓ to m is $m \cap n - \ell \cap n$, or $Q - P$, with P and Q as above. So $\sigma_m\sigma_\ell = \tau_{O,2(Q-P)} = \tau_{O, \begin{bmatrix} \frac{16}{5} \\ -\frac{8}{5} \end{bmatrix}}$.

4. Let ℓ be the line $y = 0$ (the x -axis), m the line $x = 0$ (the y -axis), n the line $y = \sqrt{3}x$. Compute the following explicitly:

a) $\sigma_n\sigma_\ell\sigma_m$

SOLUTION: $\ell \cap n = O$, and the directed angle from ℓ to n is $\frac{\pi}{3}$, so $\sigma_n\sigma_\ell = \rho_{O, \frac{2\pi}{3}} = \sigma_q\sigma_m$, where q goes through O and makes an angle of $\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$ with respect to the x -axis. So the equation of q is $y = -\frac{1}{\sqrt{3}}x$. We have

$$\sigma_n\sigma_\ell\sigma_m = \rho_{O, \frac{2\pi}{3}}\sigma_m = \sigma_q\sigma_m\sigma_m = \sigma_q.$$

b) $\sigma_n\sigma_\ell\sigma_n$

SOLUTION: As above, $\sigma_n\sigma_\ell = \rho_{O, \frac{2\pi}{3}}$. Here, we write $\rho_{O, \frac{2\pi}{3}} = \sigma_r\sigma_n$, where the directed angle from n to r is $\frac{\pi}{3}$. So r makes an angle of $\frac{2\pi}{3}$ with respect to the x -axis, hence r has the formula $y = -\sqrt{3}x$. We have

$$\sigma_n\sigma_\ell\sigma_n = \sigma_r\sigma_n\sigma_n = \sigma_r.$$