More Finite Field Problems

1. Let $f = x^3 - 3 \in \mathbb{Z}_7[x]$ and let $F = \mathbb{Z}_7[x]/(f)$, a field. Let $\alpha = [x]_f$.
   a) What are the possible orders of the elements in $F^\times$?
   b) What is the order of $\alpha$ in $F^\times$?
   c) What is the order of $-\alpha$ in $F^\times$?
   d) What is the order of $\alpha + 1$ in $F^\times$?

2. Let $f = x^6 + x + 1 \in \mathbb{Z}_2[x]$ and let $F = \mathbb{Z}_2[x]/(f)$, a field. Let $\alpha = [x]_f$.
   a) What are the possible orders of the elements in $F^\times$?
   b) What is the order of $\alpha$ in $F^\times$?
   c) What is the order of $\alpha + 1$ in $F^\times$?

3. Let $f = x^6 + x^3 + 1 \in \mathbb{Z}_2[x]$ and let $F = \mathbb{Z}_2[x]/(f)$, a field. Let $\alpha = [x]_f$.
   a) What are the possible orders of the elements in $F^\times$?
   b) What is the order of $\alpha$ in $F^\times$?
   c) What is the order of $\alpha + 1$ in $F^\times$?

4. Let $f = x^6 + x^5 + 1 \in \mathbb{Z}_2[x]$ and let $F = \mathbb{Z}_2[x]/(f)$, a field. Let $\alpha = [x]_f$.
   a) What are the possible orders of the elements in $F^\times$?
   b) What is the order of $\alpha$ in $F^\times$?
   c) What is the order of $\alpha + 1$ in $F^\times$?