1. Does the following polynomial satisfy Eisenstein’s criterion? If so, show why, and if not show why not.

\[ f = 5x^7 + 70x^3 + 60x^2 + 30 \]

2. Let \( f = 2x^5 + 2x^4 + 8x^2 + 15 \). List all the candidates for rational roots of \( f \), as identified by the theorem on rational roots.

3. Give a prime factorization in \( \mathbb{Z}_2[x] \) for the following polynomial. Give proofs that the factors are prime. (You may use the results from class on the irreducibles in degrees \( \leq 4 \)).

\[ x^7 + x^6 + x^5 + x^2 + x + 1 \]

4. Let \( f = x^4 + 4x + 8 \). In which of the following is \( f \) irreducible?
   a) \( \mathbb{Q}[x] \)  b) \( \mathbb{Z}_2[x] \)  c) \( \mathbb{Z}_3[x] \)  d) \( \mathbb{Z}_5[x] \)  e) \( \mathbb{R}[x] \)

5. Let \( f = x^4 + x^3 + 1 \in \mathbb{Z}_2[x] \), and let \( \mathbb{F} = \mathbb{Z}_2[x]/(f) \), a field. Let \( \alpha = [x]_f \).
   a) What are the possible orders of the elements of \( \mathbb{F}^\times \)?
   b) What is the order of \( \alpha \) in \( \mathbb{F}^\times \)?
   c) What is the order of \( \alpha + 1 \) in \( \mathbb{F}^\times \)?
   d) Find a primitive element of \( \mathbb{F} \).

6. Let \( f = x^2 - 3 \in \mathbb{Z}_7[x] \), and let \( \mathbb{F} = \mathbb{Z}_7[x]/(f) \), a field. Let \( \alpha = [x]_f \).
   a) What are the possible orders of the elements of \( \mathbb{F}^\times \)?
   b) What is the order of \( \alpha \) in \( \mathbb{F}^\times \)?
   c) What is the order of \( \alpha^{75} \) in \( \mathbb{F}^\times \)?
   d) What is the order of \( 3\alpha \) in \( \mathbb{F}^\times \)?
   e) What is the order of \( \alpha + 1 \) in \( \mathbb{F}^\times \)?