1. Let \( f = 7x^5 + 2x^4 + 8x^2 + 15 \). List all the candidates for rational roots of \( f \), as identified by the theorem on rational roots.

2. Give a prime factorization in \( \mathbb{Z}_2[x] \) for the following polynomial. Give proofs that the factors are prime. (You may use the results from class on the irreducibles in degrees \( \leq 3 \).)

\[ f = x^7 + x^6 + x^5 + x^4 + x^3 + x + 1 \]

3. Let \( f = x^3 + 7x + 7 \). In which of the following is \( f \) irreducible?
   a) \( \mathbb{Q}[x] \)  
   b) \( \mathbb{Z}_2[x] \)  
   c) \( \mathbb{Z}_3[x] \)  
   d) \( \mathbb{Z}_5[x] \)  
   e) \( \mathbb{R}[x] \)

4. Let \( f = x^4 + x^3 + 1 \in \mathbb{Z}_2[x] \), and let \( F = \mathbb{Z}_2[x]/(f) \), a field. Let \( \alpha = [x]_f \).
   a) What are the possible orders of the elements of \( F^\times \)?
   b) What is the order of \( \alpha \) in \( F^\times \)?
   c) What is the order of \( \alpha + 1 \) in \( F^\times \)?
   d) Find a primitive element of \( F \).

5. Let \( f = x^2 - 3 \in \mathbb{Z}_7[x] \), and let \( F = \mathbb{Z}_7[x]/(f) \), a field. Let \( \alpha = [x]_f \).
   a) What are the possible orders of the elements of \( F^\times \)?
   b) What is the order of \( \alpha \)?
   c) What is the order of \( 2\alpha \)?
   d) What is the order of \( \alpha + 1 \)?