Show all of your work.

1. What are the elements of $\mathbb{Z}_{21}^\times$?

2. What is $\phi(18,000)$?

3. a) What are the possible orders of the elements of $\mathbb{Z}_{125}^\times$?  
   b) What is the order of 21 in $\mathbb{Z}_{125}^\times$?  
   c) What is the smallest positive integer congruent to 2178252347380 mod 125?

4. What is the order of $2^{71186259842}$ in $\mathbb{Z}_{17}^\times$?

5. For which primes $p$ is $x^{41} \equiv x \mod p$ for all $x \in \mathbb{Z}$?

6. Find the smallest nonnegative solution for the following congruences.
   \[
   x \equiv 96 \mod 120  
   \]
   \[
   x \equiv 48 \mod 54  
   \]
   \[
   x \equiv 51 \mod 75  
   \]

7. Find all solutions of $x^2 \equiv 1$ in 
   a) $\mathbb{Z}_{128}$  
   b) $\mathbb{Z}_{45}$  
   c) $\mathbb{Z}_{80}$