Show all of your work.

1. What is $\phi(2700)$?

2. What is the order of $53$ in $\mathbb{Z}_n^\times$ where $n = 2^93^87^5$?

3. a) What is the smallest nonnegative integer congruent to $15^{3982702498462}$ mod $17$?

b) What is the order of $15^{3982702498462}$ in $\mathbb{Z}_{17}^\times$?

4. For which primes $p$ is $x^{17} \equiv x \mod p$ for all $x \in \mathbb{Z}$?

5. Find the smallest nonnegative solution for the following congruences.

\[
\begin{align*}
x & \equiv 21 \mod 80 \\
x & \equiv 13 \mod 28 \\
x & \equiv 11 \mod 50
\end{align*}
\]

6. Find all solutions of $x^2 \equiv 1$ in

a) $\mathbb{Z}_{64}$

b) $\mathbb{Z}_{75}$