Show all of your work.
1. What are the elements of $\mathbb{Z}_{15}\times\mathbb{Z}_{15}$?

2. What is $\phi(42,000)$?

3. a) What are the possible orders of the elements of $\mathbb{Z}_{25}\times\mathbb{Z}_{25}$?
   b) What is the order of 4 in $\mathbb{Z}_{25}\times\mathbb{Z}_{25}$?
   c) What is the smallest positive integer congruent to $4^{27915}$ mod 25?

4. What is the order of $2^{3927908}$ in $\mathbb{Z}_{13}\times\mathbb{Z}_{13}$?

5. For which primes $p$ is $x^{31} \equiv x \mod p$ for all $x \in \mathbb{Z}$?

6. Find the smallest nonnegative solution for the following congruences.
   \[
   \begin{align*}
   x &\equiv 9 \mod 80 \\
   x &\equiv 129 \mod 150 \\
   x &\equiv 39 \mod 45
   \end{align*}
   \]

7. Find all solutions of $x^2 \equiv 1$ in
   a) $\mathbb{Z}_{125}$
   b) $\mathbb{Z}_{64}$
   c) $\mathbb{Z}_{800}$