Show all of your work.

1. Prove that

\[1^3 + \cdots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2\]

for all \(n \geq 1\).

2. Use the Euclidean Algorithm to find \((4899, 2231)\), and find a solution to Bezout’s identity.

3. True or False. If false, give a counterexample.
   a) If \(ax \equiv ay \pmod{m}, a \not\equiv 0 \pmod{m}\) and \((a, x) = 1\), then \(x \equiv y \pmod{m}\).
   b) If \((m, b) = 1\), then \(ax \equiv b \pmod{m}\) has a solution for any \(a\).
   c) If \((m, a) = 1\), then \(ax \equiv b \pmod{m}\) has a solution for any \(b\).
   d) If \(ax \equiv ay \pmod{m}\), then \(x \equiv y \pmod{m/(m, a)}\).

4. Let \(a = 2^{10}3^54^25^86^3\) and \(b = 2^63^24^65^46^4\). Give prime decompositions for the following:
   a) \(ab\)
   b) \([a, b]\)
   c) \((a, b)\)

5. Find all solutions or say why none exist:
   a) \(80x + 45y = 60\)
   b) \(80x + 60y = 45\)

6. Suppose \((a, b) = 20\), \([a, b] = 1200\). What are the possible values for \(a\) and \(b\) (as a pair)?

7. Find all non-negative solutions less than the modulus or show why no solutions exist:
   a) \(36x \equiv 28 \pmod{120}\)
   b) \(28x \equiv 36 \pmod{120}\)