Show all of your work.

1. Use the Euclidean Algorithm to find \((5568, 7266)\), and find a solution to Bezout’s identity.

2. True or False. If true say why. If false, give a counterexample.
   a) \(ax \equiv b \mod 77\) has a unique solution \(\mod 77\) provided that \((a, 77)\)|\(b\).
   b) If \(a \not\equiv 0 \mod 21\), and if \(ax \equiv ay \mod 21\), then \(x \equiv y \mod \frac{21}{(21, y)}\).

3. Let \(a = 2^53^64^75^86^9\) and \(b = 2^93^84^55^63^3\). Give prime decompositions for the following:
   a) \(ab\)
   b) \([a, b]\)
   c) \((a, b)\)

4. Find all solutions or say why none exist:
   a) \(75x + 100y = 35\)
   b) \(75x + 35y = 100\)

5. Suppose \((a, b) = 40\), \([a, b] = 3600\). What are the possible values for \(a\) and \(b\) (as a pair)?

6. Find all nonnegative solutions less than the modulus or show why no solutions exist:
   a) \(24x \equiv 22 \mod 150\)
   b) \(42x \equiv 24 \mod 150\)

7. a) What are the elements of \(\mathbb{Z}_{18}^\times\)?
   b) What is the inverse of \(13\) in \(\mathbb{Z}_{18}^\times\)?