Show all of your work.

1. Prove that
\[
1^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}
\]
for all \( n \geq 1 \).

2. Use the Euclidean Algorithm to find \((7596, 2436)\), and find a solution to Bezout’s identity.

3. True or False. If false, give a counterexample.
   a) If \( a \not\equiv 0 \mod 33 \), and if \( ax \equiv ay \mod 33 \), then \( x \equiv y \mod 33/(33, x) \).
   b) If \( a|c \) and \( b|c \), then \( (ab)|c \).

4. Let \( a = 2^{10}3^24^25^76^5 \) and \( b = 2^63^34^55^46^4 \). Give prime decompositions for the following:
   a) \( ab \)
   b) \( [a, b] \)
   c) \( (a, b) \)

5. Find all solutions or say why none exist:
   a) \( 60x + 45y = 35 \)
   b) \( 60x + 35y = 45 \)

6. Suppose \((a, b) = 20\), \([a, b] = 1800\). What are the possible values for \( a \) and \( b \) (as a pair)?

7. Find all nonnegative solutions less than the modulus or show why no solutions exist:
   a) \( 68x \equiv 20 \mod 100 \)
   b) \( 20x \equiv 68 \mod 100 \)