Show all of your work.

1. Prove that
   
   \[ 1 + \cdots + n = \frac{n(n + 1)}{2} \]
   
   for all \( n \geq 1 \).

2. Use the Euclidean Algorithm to find \((10013, 3325)\), and find a solution to Bezout’s identity.

3. True or False. If false, give a counterexample.
   a) If \( a \not\equiv 0 \mod m \), and if \( ax \equiv ay \mod m \), then \( x \equiv y \mod \frac{m}{(m, x)} \).
   b) If \( a \not\equiv 0 \mod m \), and if \( ax \equiv b \mod m \) has a solution, then the solution is unique.

4. Let \( a = 2^73^24^25^76^5 \) and \( b = 2^53^34^55^46^3 \). Give prime decompositions for the following:
   a) \( ab \)
   b) \([a, b]\)
   c) \((a, b)\)

5. Find all solutions or say why none exist:
   a) \( 48x + 40y = 20 \)
   b) \( 48x + 20y = 40 \)

6. Suppose \((a, b) = 12\), \([a, b] = 1800\). What are the possible values for \(a\) and \(b\) (as a pair)?

7. Find all non-negative solutions less than the modulus or show why no solutions exist:
   a) \( 65x \equiv 15 \mod 160 \)
   b) \( 60x \equiv 10 \mod 160 \)