1. An RNA string splits as follows:
   G-fragments: AUG, UACG, UCAG, ACAAG
   U,C-fragments: GU, AC, U, C, AG, GAC, AAGAU.
   What was the original RNA string? If there is more than one possibility, give two.
   **Solution:** The abnormal fragment is AG, so that is the end of the string. We get the following directed graph:

   ![Directed Graph](attachment:image.png)

   The string ends in AG, an odd vertex in the graph. So it must begin at a different odd vertex, U. The only Euler circuit beginning at U and following the arrows is UACGACAAGAUGUCAG. So that is the original RNA string.

2. a) Is the following graph Hamiltonian? If so, find a Hamilton cycle. If not, say why not.

   ![Graph](attachment:image.png)

   **Solution:** Yes:

   ![Hamiltonian Cycle](attachment:image.png)
b) Find three nonisomorphic spanning trees in the same graph. 
Solution:

\[
\begin{align*}
&\circ\quad \circ\quad \circ \\
&\circ\quad \circ \\
&\circ\quad \circ\quad \circ \\
&\circ\quad \circ\quad \circ \\
&\circ\quad \circ\quad \circ \\
&\circ\quad \circ\quad \circ
\end{align*}
\]

c) Is the graph bipartite? If so show it by coloring the vertices. If not, say why not.
Solution: No. The graph contains a triangle. No odd cycle is bipartite. So no graph containing an odd cycle can be bipartite.
3. a) Find all nonisomorphic trees with 7 vertices.

Solution:

(1) \[ \bullet---\circ---\bullet---\circ---\bullet---\circ---\bullet \]

(2) \[ \bullet \]

(3) \[ \bullet---\circ---\bullet---\circ---\bullet---\circ---\bullet \]

(4) \[ \bullet \]

(5) \[ \bullet \]

(6) \[ \bullet \]

(7) \[ \bullet \]

(8) \[ \bullet---\circ---\bullet---\circ---\bullet \]
b) For each one, show which complete bipartite graph contains it (by coloring the vertices).

**SOLUTION:** Counting dots, we see that (1)–(5), (8) and (9) lie in $K_{4,3} \cong K_{3,4}$, while (6), (7) and (10) lie in $K_{5,2} \cong K_{2,5}$. Finally, (11) lies in $K_{6,1} \cong K_{1,6}$. 