1. Does $Q_3$ have a Hamilton cycle? If not, show why not. If so, draw one.

**Solution:** Here is $Q_3$:

And here is a Hamilton cycle in it:

2. Does $Q_3$ have a Euler circuit? Give reasons for your answer.

**Solution:** No. The vertices of $Q_3$ all have degree 3. A multi-graph has an Euler circuit if and only if every vertex has even degree.
3. Find three non-isomorphic spanning trees for $Q_3$. (Hint: count the number of vertices that have degree three in the tree.)

**SOLUTION:** We can find one by removing an edge from the Hamilton cycle in Question 1:

![Diagram of a spanning tree]

Here are two more:

![Diagram of two more spanning trees]

4. Which complete bipartite graph contains $Q_3$? Color the vertices of $Q_3$ to show it.

**SOLUTION:**

![Diagram of a complete bipartite graph]

There are four of each color, so $Q_3$ embeds in $K_{4,4}$. 
5. a) Find all non-isomorphic trees with 7 vertices.
   b) For each one, show which complete bipartite graph contains it
   (by coloring the vertices).

SOLUTION:

(1) \[ \bullet \quad \circ \quad \bullet \quad \bullet \quad \circ \quad \bullet \quad \circ \quad \bullet \]

(2) \[ \bullet \quad \circ \quad \bullet \quad \bullet \quad \circ \quad \bullet \quad \circ \quad \bullet \]

(3) \[ \bullet \quad \circ \quad \bullet \quad \bullet \quad \circ \quad \bullet \quad \circ \quad \bullet \]

(4) \[ \bullet \quad \circ \quad \bullet \quad \bullet \quad \circ \quad \bullet \quad \circ \quad \bullet \]

(5) \[ \bullet \quad \circ \quad \bullet \quad \bullet \quad \circ \quad \bullet \quad \circ \quad \bullet \]

(6) \[ \bullet \quad \circ \quad \bullet \quad \bullet \quad \circ \quad \bullet \quad \circ \quad \bullet \]

(7) \[ \bullet \quad \circ \quad \bullet \quad \bullet \quad \circ \quad \bullet \quad \circ \quad \bullet \]

(8) \[ \bullet \quad \circ \quad \bullet \quad \bullet \quad \circ \quad \bullet \quad \circ \quad \bullet \]
Counting dots, we see that (1)–(5), (8) and (9) lie in $K_{4,3} \cong K_{3,4}$, while (6), (7) and (10) lie in $K_{5,2} \cong K_{2,5}$. Finally, (11) lies in $K_{6,1} \cong K_{1,6}$. 