1. Let \( A = \begin{bmatrix}
1 & -2 & 2 & -1 & 2 & -1 & 2 \\
-2 & 5 & -7 & 0 & -3 & 2 & -3 \\
2 & -5 & 7 & 1 & 4 & -4 & 4 \\
0 & -2 & 6 & 5 & -1 & -2 & -1 \\
1 & -5 & 11 & 7 & 1 & -5 & 1 \\
\end{bmatrix} \).

Free gift: The augmented matrix \([A|b]\) reduces to
\[
\begin{bmatrix}
1 & 0 & -4 & 0 & 9 & -11 & 9 & 5b_1 + 7b_2 + 5b_3 \\
0 & 1 & -3 & 0 & 3 & -4 & 3 & 2b_1 + 3b_2 + 2b_3 \\
0 & 0 & 0 & 1 & 1 & -2 & 1 & \ b_2 + b_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4b_1 + b_2 - b_3 + b_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 5b_1 + b_2 - 2b_3 + b_5 \\
\end{bmatrix}.
\]

a) Write \( \text{Col}(A) \) as the nullspace of another matrix.
b) What are the rank and nullity of \( A \)?
c) Find a basis for \( \text{Row}(A) \).
d) Find a basis for \( \text{N}(A) \).
e) Find a basis for \( \text{Col}(A) \) consisting of columns of \( A \).
f) Write each of the columns not in the basis you gave for \( \text{Col}(A) \) as a linear combination of the basis elements.
g) What is the general solution of \( Ax = y \) for \( y = \begin{bmatrix}
-1 \\
8 \\
-10 \\
-14 \\
-23 \\
\end{bmatrix} \)?
h) With \( y \) as in part g), write \( y \) as a linear combination of the basis elements of from part e). (Hint: set the nonpivot variables equal to 0.)
i) Display an invertible matrix \( P \) with the property that \( P[A|b] \) is the displayed reduction.

2. Let \( A = \begin{bmatrix}
2 & -1 & 3 \\
1 & 0 & -2 \\
-1 & 3 & 1 \\
\end{bmatrix} \).

a) What is the first row of \( A^{\text{adj}} \)?
b) Use it to calculate \( \det A \).
3. Let $A = \begin{bmatrix} 23 & 70 & 10 \\ -9 & -26 & -2 \\ 8 & 23 & 2 \end{bmatrix}$. Then $\text{ch}_A(\lambda) = (\lambda + 2)^2(\lambda - 3)$.

a) For each eigenvalue of $A$, find a basis for the associated eigenspace.

b) Is $A$ diagonalizable? If not, say why not. If so, find a matrix $P$ such that $P^{-1}AP$ is diagonal, and display the diagonal matrix $P^{-1}AP$.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4x_1 + 3x_2 \\ 2x_1 - x_2 \end{bmatrix}$.

a) What is the matrix for $T$ with respect to the standard basis $\mathcal{E} = \{e_1, e_2\}$ of $\mathbb{R}^2$?

b) Let $\mathcal{B} = \{v_1, v_2\}$ be the basis given by $v_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$.

What is the matrix $[T]_\mathcal{B}$ of $T$ with respect to $\mathcal{B}$?

5. Let $W = \text{Span} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 2 \\ 1 \end{bmatrix}$.

a) Find an orthonormal basis for $W$.

b) Find an orthonormal basis for $W^\perp$.

c) Let $T$ be the orthogonal projection of $\mathbb{R}^3$ onto $W^\perp$. What is the matrix of $T$?

6. Let $T : P_2 \to P_2$ be given by $T(p) = p(5 - 2x)$.

a) What is the matrix of $T$ with respect to the standard basis $\mathcal{E} = \{1, x, x^2\}$?

b) What is $\text{det}(T)$?

c) What is the trace of $T$?

d) What is the characteristic polynomial $\text{ch}_T(\lambda)$?

e) Give a basis for each eigenspace of $T$.

f) Is $T$ diagonalizable? If so, find a basis $\mathcal{B}$ such that $[T]_\mathcal{B}$ is diagonal, and display the diagonal matrix $[T]_\mathcal{B}$.