1. Let \( A = \begin{bmatrix} 13 & 30 & 30 \\ -3 & -8 & -6 \\ -3 & -6 & -8 \end{bmatrix} \).

Calculation shows that \( \text{ch}_A(\lambda) = (-2 - \lambda)^2(1 - \lambda) \).

a) For each eigenvalue of \( A \), find a basis for the associated eigenspace.

b) Is \( A \) diagonalizable? If not, why not? If so, find a matrix \( P \) such that \( P^{-1}AP \) is diagonal, and display the diagonal matrix \( P^{-1}AP \).

2. Let \( A = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \)

a) For each eigenvalue of \( A \), find a basis for the associated eigenspace.

b) Is \( A \) diagonalizable? If not, why not? If so, find a matrix \( P \) such that \( P^{-1}AP \) is diagonal, and display the diagonal matrix \( P^{-1}AP \).

c) Let \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) be given by \( T(x) = Ax \) for all \( x \in \mathbb{R}^2 \). Let \( \mathcal{B} = \{v_1, v_2\} \) be the basis given by \( v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \).

What is the matrix \([T]_\mathcal{B}\) of \( T \) with respect to \( \mathcal{B} \)?

3. Let \( T : P_2 \rightarrow P_2 \) be given by \( T(p) = p(3 - 5x) \).

a) What is the matrix of \( T \) with respect to the standard basis \( \mathcal{E} = \{1, x, x^2\} \)?

b) What is \( \det(T) \)?

c) What is the trace of \( T \)?

d) What is the characteristic polynomial \( \text{ch}_T(\lambda) \)?

e) Give a basis for each eigenspace of \( T \).

f) Is \( T \) diagonalizable? If so, find a basis \( \mathcal{B} \) such that \([T]_\mathcal{B}\) is diagonal, and display the diagonal matrix \([T]_\mathcal{B}\).