Math 220 **Exam 3** **Fall '00**

1. Let $A = \begin{bmatrix} 14 & 4 \\ -25 & -6 \end{bmatrix}$.

   a) What is the characteristic polynomial $\text{ch}_A(\lambda)$?
   b) For each eigenvalue of A, find a basis for the associated eigenspace.
   c) Is A diagonalizable? If so, find a matrix P such that $P^{-1}AP$ is diagonal, and display the diagonal matrix $P^{-1}AP$.

2. Let $A = \begin{bmatrix} 10 & 0 & 9 \\ 30 & 1 & 30 \\ -12 & 0 & -11 \end{bmatrix}$. Then the characteristic polynomial of A is $(\lambda - 1)^2(\lambda + 2)$.

   a) For each eigenvalue of A, find a basis for the associated eigenspace.
   b) Is A diagonalizable? If so, find a matrix P such that $P^{-1}AP$ is diagonal, and display the diagonal matrix $P^{-1}AP$.

3. Let $T : R^2 \to R^2$ be the linear transformation
$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 - 7x_2 \\ 5x_1 + 4x_2 \end{bmatrix}.$$

   a) What is the matrix for T with respect to the standard basis $\mathcal{B} = \{e_1, e_2\}$ of R^2?
   b) Let $\mathcal{B}' = \{\mathbf{v}_2, \mathbf{v}_2\}$ be the basis given by $\mathbf{v}_1 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. What is the matrix $[T]_{\mathcal{B}'}$ of T with respect to $\mathcal{B}'$? (Use the transition matrix $P = [I]_{\mathcal{B},\mathcal{B}'}$ from $\mathcal{B}'$ to $\mathcal{B}$ to find it.)

4. Let $T : P_2 \to P_2$ be given by $T(p) = p(3 - 2x)$.

   a) What is the matrix of T with respect to the standard basis $\mathcal{B} = \{1, x, x^2\}$?
   b) What is $\det(T)$?
   c) What are the rank and nullity of T?
   d) What is the characteristic polynomial $\text{ch}_T(\lambda)$?
   e) What are the eigenvalues of T?
   f) Is T diagonalizable?