1. Let \( A = \begin{bmatrix} 14 & 4 \\ -25 & -6 \end{bmatrix} \).

a) What is the characteristic polynomial \( \chi_A(\lambda) \)?

b) For each eigenvalue of \( A \), find a basis for the associated eigenspace.

c) Is \( A \) diagonalizable? If so, find a matrix \( P \) such that \( P^{-1}AP \)

is diagonal, and display the diagonal matrix \( P^{-1}AP \).

2. Let \( A = \begin{bmatrix} 10 & 0 & 9 \\ 30 & 1 & 30 \\ -12 & 0 & -11 \end{bmatrix} \). Then the characteristic polynomial

of \( A \) is \( (\lambda - 1)^2(\lambda + 2) \).

a) For each eigenvalue of \( A \), find a basis for the associated eigenspace.

b) Is \( A \) diagonalizable? If so, find a matrix \( P \) such that \( P^{-1}AP \)

is diagonal, and display the diagonal matrix \( P^{-1}AP \).

3. Let \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) be the linear transformation

\[ T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} 3x_1 - 7x_2 \\ 5x_1 + 4x_2 \end{bmatrix}. \]

a) What is the matrix for \( T \) with respect to the standard basis \( B = \{ e_1, e_2 \} \) of \( \mathbb{R}^2 \)?

b) Let \( B' = \{ v_2, v_2 \} \) be the basis given by \( v_1 = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \) and \( v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \).

What is the matrix \( [T]_{B'} \) of \( T \) with respect to \( B' \)? (Use the

transition matrix \( P = [I]_{B,B'} \) from \( B' \) to \( B \) to find it.)

4. Let \( T : P_2 \to P_2 \) be given by \( T(p) = p(3-2x) \).

a) What is the matrix of \( T \) with respect to the standard basis \( B = \{ 1, x, x^2 \} \)?

b) What is \( \det(T) \)?

c) What are the rank and nullity of \( T \)?

d) What is the characteristic polynomial \( \chi_T(\lambda) \)?

e) What are the eigenvalues of \( T \)?

f) Is \( T \) diagonalizable?