

1. The minitab worksheet [e3-1](#) gives scores on two standardized tests taken by the same individuals. The second test covers more advanced material, and is supposed to be harder. You wish to see if this is true, at least on average.
 - a) What are t and P for this study? What do you conclude?

SOLUTION: We are studying two variables associated with a single sample, so we can treat the difference of the two variables as a variable in its own right.

Thus, we set $C3 = C1 - C2$ in minitab. The alternative hypothesis is that the average for the first test should be higher than the average for the second, i.e., that the average for the population represented by C3 should be positive. The null hypothesis is that the average for the population represented by C3 is 0. It is a one-tailed test.

The basic statistics for C3 has $n = 25$, $\bar{x} = 6.68$, and $s = 13.89$. Thus,

$$t = \frac{\bar{x} - 0}{\left(\frac{s}{\sqrt{n}}\right)} \approx 2.405$$

There are 24 degrees of freedom, so P is between 1% and 2%. The test is significant, and supports the alternative hypothesis.

- b) Give a 95% confidence interval for the difference in the means for the two tests.

SOLUTION: The radius of a 95% confidence interval is t^* times the standard error, where t^* is the entry on the t whose tail size is 2.5%. Thus, $t^* = 2.064$. The standard error is $s/\sqrt{n} \approx 2.778$. Thus, the confidence interval is

$$6.68 \pm 5.734$$

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2. A new treatment for high blood pressure is tested. The patients are randomly divided into two groups. One group gets the new treatment, and the other gets the old treatment. The results are as follows. Here \bar{x} is the average systolic blood pressure for each group:

	n	\bar{x}	s
New treatment group	240	126	25
Old treatment group	235	130	23

- a) You wish to show that the new treatment lowers blood pressure more than the old one. So what are your null and alternative hypotheses?

SOLUTION: The alternative hypothesis is that the average for the new treatment is lower than the average for the old treatment. The null hypothesis is that the two averages are the same. It is a one-tailed test.

- b) What are the values of t and P ? What do you conclude?

SOLUTION: Here,

$$t = \frac{(126 - 130) - 0}{\sqrt{\frac{25^2}{240} + \frac{23^2}{235}}} \approx -1.815$$

There are 234 degrees of freedom, which lies between the 100 and 1000 lines on the t table. Since it is closer to 100, we use that line.

Since the t curve is symmetric, we can use the absolute value of t , giving P between 2.5% and 5%. The test is significant, and supports the alternative hypothesis.

- c) Give a 90% confidence interval for the difference in the average blood pressures for patients under the two treatments.

SOLUTION: Here, $t^* = 1.660$ is obtained from the t table by setting the tail size to 5%. The radius of the confidence interval is

$$1.660 \cdot \sqrt{\frac{25^2}{240} + \frac{23^2}{235}} \approx 3.658,$$

so the confidence interval is -4 ± 3.658 (if we subtract the average for the old treatment for the average for the new).

3. A new procedure is developed to try to lower the standard deviation for a certain variable. Under the standard procedure, we have $\sigma = 40$. A test of the new procedure gives $s = 30$ with a sample size of 16.

a) What are χ^2 and P?

SOLUTION: Here,

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = 15 \cdot (30/40)^2 = 8.4375$$

There are 15 degrees of freedom, so P is between 5% and 10%. The test is inconclusive.

- b) Give a 90% confidence interval for the standard deviation under the new procedure.

SOLUTION: With 15 degrees of freedom, a 90% confidence interval for χ^2 is given by $7.26 < \chi^2 < 25$. Substituting the formula for χ^2 , we get

$$7.26 < \frac{15 \cdot 30^2}{\sigma^2} < 25$$

Noting that $15 \cdot 30^2 = 13500$, and inverting, we get

$$\frac{1}{7.26} > \frac{\sigma^2}{13500} > \frac{1}{25}$$

Multiplying through by 13500 and taking square roots, we see

$$\sqrt{\frac{13500}{7.26}} > \sigma > \sqrt{\frac{13500}{25}}$$

Evaluating the roots and writing the terms in ascending order, we get

$$23.24 < \sigma < 43.12$$

4. A die is tested for fairness. Here are the results:

number on die	1	2	3	4	5	6
observed freq.	45	32	37	46	36	44

What are χ^2 and P? What do you conclude?

SOLUTION: The fairness hypothesis says we expect to get the same number of occurrences for each number on the die. Since the sample size is 240, we expect $240/6 = 40$ occurrences for each number. Thus,

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$$\begin{aligned}\chi^2 &= \frac{(45 - 40)^2}{40} + \frac{(32 - 40)^2}{40} + \frac{(37 - 40)^2}{40} + \frac{(46 - 40)^2}{40} \\ &\quad + \frac{(36 - 40)^2}{40} + \frac{(44 - 40)^2}{40} \\ &\approx 4.15\end{aligned}$$

There are six categories of data (the numbered faces), so there are five degrees of freedom for this test. We see that P is between 30% and 50%, so the test does not rule out fairness.

5. A survey is done to see if voting and gender are independent. The results are as follows:

	Men	Women
Voted	40	64
Didn't vote	160	130

What are χ^2 and P? What do you conclude?

SOLUTION: The table of totals is

	Men	Women	Total
Voted	40	64	104
Didn't vote	160	130	290
Total	200	194	394

Thus, the expected frequencies are

	Men	Women
Voted	$\frac{104 \cdot 200}{394} \approx 52.79$	$\frac{104 \cdot 194}{394} \approx 51.21$
Didn't vote	$\frac{290 \cdot 200}{394} \approx 147.21$	$\frac{290 \cdot 194}{394} \approx 142.79$

Thus,

$$\begin{aligned}\chi^2 &= \frac{(40 - 52.79)^2}{52.79} + \frac{(64 - 51.21)^2}{51.21} + \frac{(160 - 147.21)^2}{147.21} \\ &\quad + \frac{(130 - 142.79)^2}{142.79} \\ &\approx 8.55\end{aligned}$$

There are $(2 - 1) \cdot (2 - 1) = 1$ degrees of freedom, so $P < 1\%$. Thus, the test shows that gender and voting are not likely to be independent.