Math 108 **Exam 3 Solutions** **Fall '02**

1. A study is done of the weight of male students at the University at Albany. With a sample size of 21, the sample average is 165 lb., and the sample SD^+ is 25. Give 80% confidence intervals for
   a) the average
   b) the standard deviation
   for the weight of male students at Albany.

   Solution: We first compute the standard error:

$$\text{SE} = \frac{\text{SD}^+}{\sqrt{21}} = \frac{25}{\sqrt{21}} \approx 5.46 \,.$$

   We now go to the t-table to compute the radius of an 80% confidence interval for the average. With 20 degrees of freedom, the radius of the interval is 1.33 SE, which comes to about 7.26. So the interval is

$$(165 - 7.26,\ 165 + 7.26) = (157.74,\ 172.26).$$

   To find the confidence interval for the SD, we first go to the χ^2 table. With 20 degrees of freedom, an 80% confidence interval will have χ^2 values between 12.44 and 28.41. We set $\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{20\cdot 25^2}{\sigma^2} = \frac{12500}{\sigma^2}$. Thus, the interval is given by

$$12.44 < \frac{12500}{\sigma^2} < 28.41$$

$$\frac{1}{28.41} < \frac{\sigma^2}{12500} < \frac{1}{12.44}$$

$$\sqrt{\frac{12500}{28.41}} < \sigma < \sqrt{\frac{12500}{12.44}}.$$

   Evaluating the square roots, this gives a confidence interval of (20.98, 31.70).

2. The average weight of American men may be taken to be 175, with an SD of 30. Us the data from the last problem to test the hypothesis that male Albany students are lighter on average than American men as a whole. What are the values of t, df, and P? What do you conclude?

   SOLUTION: Here, the SD is given by the null hypothesis: 30. So the SE is $\frac{30}{\sqrt{21}} \approx 6.55$. Thus,

$$t = \frac{165 - 175}{6.55} \approx 1.53.$$

   With 20 degrees of freedom, this gives a P-value between 5% and 10%, so the test is inconclusive.

3. A poll is taken in two different towns regarding a ballot proposition. In each town, 400 people are polled. In Town A, 65% of the sample support the proposition. In Town B, 71% of the sample support the proposition. Is the difference between the two towns real, or is it due to chance error? What are z and P? What do you conclude?

   SOLUTION: Here we have two samples. The standard error for the first one is $\frac{\sqrt{.65 \cdot .35}}{\sqrt{400}}$, and the SE for the second is $\frac{\sqrt{.71 \cdot .29}}{\sqrt{400}}$. So the SE for the difference is

$$\text{SE} = \sqrt{\left(\frac{\sqrt{.65 \cdot .35}}{\sqrt{400}}\right)^2 + \left(\frac{\sqrt{.71 \cdot .29}}{\sqrt{400}}\right)^2} \approx 3.29\%$$

   This gives

$$z \approx \frac{(.71 - .65) - 0}{.0329} \approx 1.82.$$

   This gives a P-value of about 3.44%, so the test is significant. The difference between the two towns is likely to be real.

4. A die is tested for fairness, rolling it 120 times. Here are the results:

number on die	1	2	3	4	5	6
observed freq.	27	15	24	25	14	15

What are χ^2, df, and P? What do you conclude?

SOLUTION: The null hypothesis is that the die is fair, so we expect the same frequency for each number on the die. Thus, the expected frequency is 20 for each. The value of χ^2 is the sum of all the terms $\frac{(\text{obs.}-\text{exp.})^2}{\text{exp.}}$. Thus,

$$\chi^2 = \frac{(27-20)^2}{20} + \frac{(15-20)^2}{20} + \frac{(24-20)^2}{20} + \frac{(25-20)^2}{20} + \frac{(14-20)^2}{20} + \frac{(15-20)^2}{20}$$
$$= \frac{49+25+16+25+36+25}{20} = 8.8.$$

There are six categories here (for the six numbers on the die), hence five degrees of freedom. So the χ^2 table gives a P-value between 10% and 30%, so the test is inconclusive.

5. Test results are given for a second die, but you suspect the tester of fudging the results. The results presented are as follows:

number on die	1	2	3	4	5	6
observed freq.	21	19	22	19	19	20

What are χ^2, df, and P? What do you conclude?

SOLUTION: We calculate χ^2 as in the last problem:

$$\chi^2 = \frac{1}{20} + \frac{1}{20} + \frac{4}{20} + \frac{1}{20} + \frac{1}{20} + \frac{0}{20} = .4\,.$$

With 5 degrees of freedom, this is on the far left side of the curve, with a P-value less than 1%. Thus, the observed frequencies are improbably close to the expected ones, strongly suggesting that the data is fudged.

6. A study is made to see if voting and gender are independent. 200 men and 300 women are polled, to find out if they voted in the last election. The data obtained were as follows.

	Men	Women	Total
Voted	95	165	260
Didn't vote	105	135	240
Total	200	300	500

What are the values of χ^2, df, and P? What do you conclude?

SOLUTION: 2/5 of the people surveyed are men and 3/5 are women. Thus, we expect 2/5 of the 260 voters to be men and 3/5 of them to be women. Similarly, we expect 2/5 of the 240 who didn't vote to be men and 3/5 to be women. So our expected frequencies are:

	Men	Women
Voted	$.4 \cdot 260 = 104$	$.6 \cdot 260 = 156$
Didn't vote	$.4 \cdot 240 = 96$	$.6 \cdot 240 = 144$

Thus,

$$\chi^4 = \frac{(95-104)^2}{104} + \frac{(165-156)^2}{156} + \frac{(105-96)^2}{96} + \frac{(135-144)^2}{144}$$
$$= 81 \cdot \left(\frac{1}{104} + \frac{1}{156} + \frac{1}{96} + \frac{1}{144}\right) \approx 2.70.$$

There are $(2-1)\cdot(2-1) = 1$ degrees of freedom, so P is between 10% and 30%. The test is inconclusive.

7. A new procedure is developed to try to lower the standard deviation for a certain variable. Under the standard procedure, we have $\sigma = 30$. A test of the new procedure gives $S = 21$ with a sample size of 17. What are χ^2, df, and P? What do you conclude?

SOLUTION: $\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{16\cdot 21^2}{30^2} = 7.84$. Since df $= 16$, this gives a P-value between 1% and 5%, so the test is significant. The new procedure likely lowers the SD.