Math 108 **Exam 2 Solutions** **Fall '02**

1. Deal two cards from a well shuffled deck.

   a) What is the probability the first is an king or the second is a queen?

   SOLUTION:

$$
\begin{aligned}
P(\text{1st K or 2nd Q}) &= P(\text{1st K}) + P(\text{2nd Q}) - P(\text{1st K and 2nd Q}) \\
&= P(\text{1st K}) + P(\text{2nd Q}) \\
&\qquad - P(\text{2nd Q} \mid \text{1st K})P(\text{1st K}) \\
&= \frac{4}{52} + \frac{4}{52} - \frac{4}{51}\frac{4}{52}
\end{aligned}
$$

   b) What is the probability both are kings?

   SOLUTION: $P(\text{both K}) = P(\text{2nd K} \mid \text{1st K})P(\text{1st K}) = \frac{3}{51}\frac{4}{52}$.

   c) What is the probability at least one of them is a king?

   SOLUTION:

$$
\begin{aligned}
P(\text{at least one K}) &= P(\text{1st K}) + P(\text{2nd K}) - P(\text{both K}) \\
&= \frac{4}{52} + \frac{4}{52} - \frac{3}{51}\frac{4}{52}
\end{aligned}
$$

2. Roll a pair of dice. What is the probability you got doubles given that the sum is 4?

   SOLUTION: There are 3 ways to get a sum of 4: $(1,3)$, $(2,2)$, $(3,1)$. One of the three is "doubles". So the conditional probability is $\frac{1}{3}$.

3. An urn contains 7 red balls and 9 green ones. Draw six times with replacement. What is the probability you get more green balls than red ones?

   SOLUTION: To get more green than red, you must get at least 4 green.

$$
\begin{aligned}
P(\geq 4\text{ G}) &= P(4\text{ G}) + P(5\text{ G}) + P(6\text{ G}) \\
&= \binom{6}{4}\left(\frac{9}{16}\right)^4\left(\frac{7}{16}\right)^2 + \binom{6}{5}\left(\frac{9}{16}\right)^5\left(\frac{7}{16}\right)^1 \\
&\qquad + \binom{6}{6}\left(\frac{9}{16}\right)^6\left(\frac{7}{16}\right)^0.
\end{aligned}
$$

4. The probability of winning roulette is $\frac{1}{38}$. Play roulette 50 times. What is the probability you win at least twice?

   Solution:

$$\begin{aligned} P(\geq 2\text{ W}) &= 1 - P(< 2\text{ W}) = 1 - [P(0\text{ W}) + P(1\text{ W})] \\ &= 1 - \left[\left(\frac{37}{38}\right)^{50} + \binom{50}{1}\left(\frac{1}{38}\right)^{1}\left(\frac{37}{38}\right)^{49}\right]. \end{aligned}$$

5. A true-false test has 25 questions. A correct answer is worth 2 points. If you answer incorrectly, you lose a point. The passing score is 25. What is the probability you pass by guessing?

   Solution: Guessing an answer is like drawing at random from the box

$$\boxed{\boxed{2}\ \ \boxed{\text{-1}}}.$$

   Here, avg(box) $= \frac{1}{2}$, and SD(box) $= (2-(-1))\sqrt{\frac{1}{2}\cdot\frac{1}{2}} = 3\cdot\frac{1}{2} = \frac{3}{2}$. With 25 draws, we have

$$\begin{aligned} \text{EV} &= 25 \cdot \text{avg(box)} = 12.5 \\ \text{SE} &= \sqrt{25}\cdot\text{SD(box)} = 7.5\ . \end{aligned}$$

   We now find the probability you get a 25 or better by guessing:

$$\text{SU}(25) = \frac{25 - \text{EV}}{\text{SE}} = \frac{25 - 12.5}{7.5} \approx 1.67\ .$$

   We want the area under the normal curve above $z = 1.67$, which is also the area below $z = -1.67$. The normal table gives this as 4.75%. Thus, the probability you pass by guessing is 4.75%.

6. Game: Draw twice with replacement from the following box:

$$\boxed{\boxed{1}\,\boxed{2}\,\boxed{3}\,\boxed{4}\,\boxed{5}\,\boxed{6}\,\boxed{7}\,\boxed{8}}$$

   If the sum is 4, you win \$19. Otherwise you lose \$1.

   Suppose you play the game 400 times. The next three questions concern your net gain/loss.
   a) What is the probability you lose at least \$20?
   b) What is the probability you lose at least \$100?
   c) What is the probability you win at least \$100?

   Solution: Out of the 64 ways to draw twice with replacement from an 8-card box, 3 ways have a sum of 4: (3,1), (2,2) and (1,3). Thus, there are 3 winning tickets and 61 losing tickets. So the payoff from playing the game is like drawing with replacement from the box

3 [19] 61 [-1]

and adding the numbers drawn. In this case,

$$\text{avg(box)} = \frac{3 \cdot 19 + 61 \cdot (-1)}{64} = -\frac{4}{64} = -\frac{1}{16}$$

$$\text{SD(box)} = (19 - (-1))\sqrt{\frac{3}{64} \cdot \frac{61}{64}} \approx 4.227 \ .$$

With 400 draws from the box, we have

$$\text{EV} = 400 \cdot \text{avg(box)} = 400(-\frac{1}{16}) = -25$$

$$\text{SE} = \sqrt{400} \cdot \text{SD(box)} \approx 20 \cdot 4.227 \approx 84.55 \ .$$

a) For the probability of losing at least \$20, we have

$$\text{SU}(-20) = \frac{-20 - \text{EV}}{\text{SE}} \approx \frac{-20 - (-25)}{84.55} \approx .06 \ .$$

We want the area under the normal curve below this, which comes directly from the normal table: the probability is 52.39%.

b) For the probability of losing at least \$100, we have

$$\text{SU}(-100) = \frac{-100 - \text{EV}}{\text{SE}} \approx \frac{-100 - (-25)}{84.55} \approx -.89 \ .$$

Again, we want the area under the normal curve below this: the probability is 18.67%.

c) For the probability of winning at least \$100, we have

$$\text{SU}(100) = \frac{100 - \text{EV}}{\text{SE}} \approx \frac{100 - (-25)}{84.55} \approx 1.48 \ .$$

This time we want the area above it, which is the same as the area below -1.48. From the normal table, we see the probability is 6.94%.