1. Deal two cards from a well shuffled deck.
   a) What is the probability the first is an king or the second is a queen?

   Solution:
   \[ P(1\text{st }K \text{ or } 2\text{nd }Q) = P(1\text{st }K) + P(2\text{nd }Q) - P(1\text{st }K \text{ and } 2\text{nd }Q) \]
   \[ = P(1\text{st }K) + P(2\text{nd }Q) - P(2\text{nd }Q | 1\text{st }K) P(1\text{st }K) \]
   \[ = \frac{4}{52} + \frac{4}{52} - \frac{4}{51} \cdot \frac{4}{52} \]

   b) What is the probability both are kings?

   Solution: \( P(\text{both }K) = P(2\text{nd }K | 1\text{st }K) P(1\text{st }K) = \frac{3}{51} \cdot \frac{4}{52} \).

   c) What is the probability at least one of them is a king?

   Solution:
   \[ P(\text{at least one }K) = P(1\text{st }K) + P(2\text{nd }K) - P(\text{both }K) \]
   \[ = \frac{4}{52} + \frac{4}{52} - \frac{3}{51} \cdot \frac{4}{52} \]

2. Roll a pair of dice. What is the probability you got doubles given that the sum is 4?

   Solution: There are 3 ways to get a sum of 4: (1, 3), (2, 2), (3, 1). One of the three is “doubles”. So the conditional probability is \( \frac{1}{3} \).

3. An urn contains 7 red balls and 9 green ones. Draw six times with replacement. What is the probability you get more green balls than red ones?

   Solution: To get more green than red, you must get at least 4 green.
   \[ P(\geq 4 \text{ G}) = P(4 \text{ G}) + P(5 \text{ G}) + P(6 \text{ G}) \]
   \[ = \binom{6}{4} \left( \frac{9}{16} \right)^4 \left( \frac{7}{16} \right)^2 + \binom{6}{5} \left( \frac{9}{16} \right)^5 \left( \frac{7}{16} \right)^1 \]
   \[ + \binom{6}{6} \left( \frac{9}{16} \right)^6 \left( \frac{7}{16} \right)^0 . \]
4. The probability of winning roulette is \(\frac{1}{38}\). Play roulette 50 times. What is the probability you win at least twice?

**Solution:**

\[
P(\geq 2 \text{ W}) = 1 - P(<2 \text{ W}) = 1 - [P(0 \text{ W}) + P(1 \text{ W})]
\]

\[
= 1 - \left[ \left( \frac{37}{38} \right)^{50} + \binom{50}{1} \left( \frac{1}{38} \right) \left( \frac{37}{38} \right)^{49} \right].
\]

5. A true-false test has 25 questions. A correct answer is worth 2 points. If you answer incorrectly, you lose a point. The passing score is 25. What is the probability you pass by guessing?

**Solution:** Guessing an answer is like drawing at random from the box 

\[
\begin{bmatrix}
2 \\ -1
\end{bmatrix}
\]

Here, \(\text{avg}(\text{box}) = \frac{1}{2}\), and \(\text{SD}(\text{box}) = (2 - (-1))\sqrt{\frac{1}{2} \cdot \frac{1}{2}} = 3 \cdot \frac{1}{2} = \frac{3}{2}\).

With 25 draws, we have

\[
\text{EV} = 25 \cdot \text{avg}(\text{box}) = 12.5
\]

\[
\text{SE} = \sqrt{25} \cdot \text{SD}(\text{box}) = 7.5.
\]

We now find the probability you get a 25 or better by guessing:

\[
\text{SU}(25) = \frac{25 - \text{EV}}{\text{SE}} = \frac{25 - 12.5}{7.5} \approx 1.67.
\]

We want the area under the normal curve above \(z = 1.67\), which is also the area below \(z = -1.67\). The normal table gives this as 4.75%. Thus, the probability you pass by guessing is 4.75%.

6. **Game:** Draw twice with replacement from the following box:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8
\end{bmatrix}
\]

If the sum is 4, you win $19. Otherwise you lose $1.

Suppose you play the game 400 times. The next three questions concern your net gain/loss.

a) What is the probability you lose at least $20?

b) What is the probability you lose at least $100?

c) What is the probability you win at least $100?

**Solution:** Out of the 64 ways to draw twice with replacement from an 8-card box, 3 ways have a sum of 4: (3,1), (2,2) and (1,3). Thus, there are 3 winning tickets and 61 losing tickets. So
the payoff from playing the game is like drawing with replacement from the box

\begin{array}{ccc}
3 & 19 & 61 \\
\end{array}

and adding the numbers drawn. In this case,

\[
\text{avg}(\text{box}) = \frac{3 \cdot 19 + 61 \cdot (-1)}{64} = -\frac{4}{64} = -\frac{1}{16}
\]

\[
\text{SD}(\text{box}) = (19 - (-1))\sqrt{\frac{3 \cdot 61}{64}} \approx 4.227
\]

With 400 draws from the box, we have

\[
\text{EV} = 400 \cdot \text{avg}(\text{box}) = 400(-\frac{1}{16}) = -25
\]

\[
\text{SE} = \sqrt{400 \cdot \text{SD}(\text{box})} \approx 20 \cdot 4.227 \approx 84.55
\]

a) For the probability of losing at least $20, we have

\[
\text{SU}(-20) = \frac{-20 - \text{EV}}{\text{SE}} \approx \frac{-20 - (-25)}{84.55} \approx .06
\]

We want the area under the normal curve below this, which comes directly from the normal table: the probability is 52.39%.

b) For the probability of losing at least $100, we have

\[
\text{SU}(-100) = \frac{-100 - \text{EV}}{\text{SE}} \approx \frac{-100 - (-25)}{84.55} \approx -.89
\]

Again, we want the area under the normal curve below this: the probability is 18.67%.

c) For the probability of winning at least $100, we have

\[
\text{SU}(100) = \frac{100 - \text{EV}}{\text{SE}} \approx \frac{100 - (-25)}{84.55} \approx 1.48
\]

This time we want the area above it, which is the same as the area below $-1.48$. From the normal table, we see the probability is 6.94%.