Section 18: confidence interval & hypothesis testing using sample means (sigma unknown)

1. A simple random sample of 40 packages of Chips Ahoy cookies reveals an average of 1261.6 chocolate chips per package, with a standard deviation of 117.6 chips. (a) Find 95%, 90%, and 99% confidence intervals for the mean number of chocolate chips in all Chips Ahoy packages.

$$df = n - 1 = 39 \qquad \text{95\% confidence: } t = 2.023$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 1261.6 \pm 2.023\,\frac{117.6}{\sqrt{40}} = 1261.6 \pm 37.62$$

$$(1223.98, 1299.22)$$

$$\text{90\% confidence: } t = 1.685$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 1261.6 \pm 1.685\,\frac{117.6}{\sqrt{40}} = 1261.6 \pm 31.33$$

$$(1230.27, 1292.93)$$

$$\text{99\% confidence: } t = 2.708$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 1261.6 \pm 2.708\,\frac{117.6}{\sqrt{40}} = 1261.6 \pm 50.35$$

$$(1292.93, 1311.95)$$

(b) The company claims that Chips Ahoy packages contain on average 1300 chocolate chips. Do we have evidence at the 10%, 5%, 1% levels that they're exaggerating?

$$H_0: \mu = 1300$$

$$H_a: \mu < 1300$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1261.6 - 1300}{\frac{117.6}{\sqrt{40}}} = -2.065$$

$$\text{1 tail:} \quad 2.426 > 2.065 > 2.023$$

$$\text{p} - \text{value: } 0.01 < P < 0.025 \qquad \mathit{Thus}:$$

$$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$P < .05 \qquad \text{reject } H_0, \text{evidence of } H_a$

$P \not< .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$

(c) Jack claims that Chips Ahoy packages contain on average 1300 chocolate chips. Do we have evidence at the 10%, 5%, 1% levels that he is mistaken?

$$H_a: \mu \neq 1300$$

$$\text{2 tail:} \quad 2.426 > 2.065 > 2.023$$

$$\text{p} - \text{value:} \;\; 0.02 < P < 0.05 \qquad Thus:$$

$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$

$P < .05 \qquad \text{reject } H_0, \text{evidence of } H_a$

$P \not< .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$

2. How many licks does it take to get to the Tootsie Roll center of a Tootsie Pop? A simple random sample of 22 attempts yields an average of 508 licks per pop, with a standard deviation of 164 licks. (a) Find 95%, 90%, and 99% confidence intervals for the mean number of licks for all pops.

$$df = n - 1 = 21 \qquad \text{95\% confidence:} \;\; t = 2.080$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 508 \pm 2.080\,\frac{164}{\sqrt{22}} = 508 \pm 72.73$$

$$(435.27, 580.73)$$

$$\text{90\% confidence:} \;\; t = 1.721$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 508 \pm 1.721\,\frac{164}{\sqrt{22}} = 508 \pm 60.17$$

$$(447.83, 568.17)$$

$$\text{99\% confidence:} \;\; t = 2.831$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 508 \pm 2.831\,\frac{164}{\sqrt{22}} = 508 \pm 98.99$$

$$(409.01, 606.99)$$

(b) Mr. Owl claims that it takes on average 500 licks. Do we have evidence at the 10%, 5%, 1% levels that he's underestimating? Do we have evidence at the 10%, 5%, 1% levels that he's mistaken?

$$H_0: \mu = 500$$

$$H_a: \mu > 500$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{508 - 500}{\frac{164}{\sqrt{22}}} = 0.229$$

$$\text{1 tail:} \quad 0.686 > 0.229$$

$$\text{p} - \text{value:} \quad 0.25 < P \qquad \text{Thus:}$$

$$P \nless .10 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .05 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$H_a: \mu \neq 500$$

$$\text{2 tail:} \quad 0.686 > 0.229$$

$$\text{p} - \text{value:} \quad 0.50 < P \qquad \text{Thus:}$$

$$P \nless .10 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .05 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

(c) Mr. Turtle claims that it takes on average 580 licks. Do we have evidence at the 10%, 5%, 1% levels that he's overestimating? Do we have evidence at the 10%, 5%, 1% levels that he's mistaken?

$$H_0: \mu = 580$$

$$H_a: \mu < 580$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{508 - 580}{\frac{164}{\sqrt{22}}} = -2.059$$

$$1\text{ tail: } \quad 2.080 > 2.059 > 1.721$$

$$\text{p} - \text{value: } \quad 0.025 < P < 0.05 \qquad \text{Thus:}$$

$$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P < .05 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$H_a: \mu \neq 580$$

$$2\text{ tail: } \quad 2.080 > 2.059 > 1.721$$

$$\text{p} - \text{value: } \quad 0.05 < P < 0.10 \qquad \text{Thus:}$$

$$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P \nless .05 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

3. What is the average commute time for people going to work in the northeast? A simple random sample of 30 people who drive to work in the northeast results in a mean time of 27.97 minutes with a standard deviation of 10.04 minutes. (a) Find 80%, 98%, and 95% confidence intervals for the average commute time in the northeast.

$$df = n - 1 = 29 \qquad 80\% \text{ confidence: } t = 1.311$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 27.97 \pm 1.311\,\frac{10.04}{\sqrt{30}} = 27.97 \pm 2.40$$

$$(25.57, 30.37)$$

$$98\% \text{ confidence: } t = 2.462$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 27.97 \pm 2.462\,\frac{10.04}{\sqrt{30}} = 27.97 \pm 5.51$$

$$(23.46, 32.48)$$

$$95\% \text{ confidence: } t = 2.045$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 27.97 \pm 2.045\,\frac{10.04}{\sqrt{30}} = 27.97 \pm 4.51$$

$$(24.22, 31.72)$$

(b) Jack claims that on average in takes 32 minutes for someone in the northeast to get to work. Do we have evidence at the 10%, 5%, 1% levels that he's overestimating? Do we have evidence at the 10%, 5%, 1% levels that he's mistaken?

$$H_0: \mu = 32$$

$$H_a: \mu < 32$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{27.97 - 32}{\frac{10.04}{\sqrt{30}}} = -2.199$$

$$\text{1 tail:} \quad 2.462 > 2.199 > 2.045$$

$$\text{p − value:} \quad 0.01 < P < 0.025 \qquad \text{Thus:}$$

$$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P < .05 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$H_a: \mu \neq 32$$

$$\text{2 tail:} \quad 2.462 > 2.199 > 2.045$$

$$\text{p − value:} \quad 0.02 < P < 0.05 \qquad \text{Thus:}$$

$$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P < .05 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

(c) Sam claims that on average in takes 25 minutes for someone in the northeast to get to work. Do we have evidence at the 10%, 5%, 1% levels that he's underestimating? Do we have evidence at the 10%, 5%, 1% levels that he's mistaken?

$$H_0: \mu = 25$$

$$H_a: \mu > 25$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{27.97 - 25}{\frac{10.04}{\sqrt{30}}} = 1.620$$

$$1 \text{ tail:} \quad 1.699 > 1.620 > 1.479$$

$$\text{p} - \text{value:} \;\; 0.05 < P < 0.075 \qquad \text{Thus:}$$

$$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P \nless .05 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$H_a: \mu \neq 25$$

$$2 \text{ tail:} \quad 1.699 > 1.620 > 1.479$$

$$\text{p} - \text{value:} \;\; 0.10 < P < 0.15 \qquad \text{Thus:}$$

$$P \nless .10 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .05 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

4. Jonathan Kent has been using a certain brand of chicken feed for years, and on average his chickens weigh 62.2 ounces. He tests a new type of chicken feed on 9 chickens, and their average weight is 64.38 ounces, with a standard deviation of 2.065 ounces. (a) Find 99%, 95%, and 85% confidence intervals for the average weight that would be gained by all chickens using the new feed.

$$df = n - 1 = 8 \qquad 99\% \text{ confidence: } \; t = 3.355$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 64.38 \pm 3.355\,\frac{2.065}{\sqrt{9}} = 64.38 \pm 2.31$$

$$(62.07, 66.69)$$

$$95\% \text{ confidence: } \; t = 2.306$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 64.38 \pm 2.306\,\frac{2.065}{\sqrt{9}} = 64.38 \pm 1.59$$

$$(62.79, 65.97)$$

$$85\% \text{ confidence: } t = 1.592$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 64.38 \pm 1.592\,\frac{2.065}{\sqrt{9}} = 64.38 \pm 1.10$$

$$(63.28, 65.48)$$

(b) Does Jonathan have evidence at the 10%, 5%, 1% levels that the new chicken feed is better than the old brand?

$$H_0: \mu = 62.2$$

$$H_a: \mu > 62.2$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{64.38 - 62.2}{\frac{2.065}{\sqrt{9}}} = 3.167$$

$$1 \text{ tail: } \quad 3.355 > 3.167 > 2.896$$

$$\text{p} - \text{value: } \; 0.005 < P < 0.01 \qquad \text{Thus:}$$

$$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P < .05 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P < .01 \qquad \text{reject } H_0, \text{evidence of } H_a$$

5. A certain comedian measures the effectiveness of his routine by the number of times he has to wait for laughter to subside before continuing; his old routine averaged 63.2 times. He is trying a new routine, and of its 15 performances, the average number of times he had to wait was 69.53 times, with a standard deviation of 19.3 times. (a) Find 99%, 95%, 90% confidence intervals for the average number of waiting times for all possible performance of his new routine (i.e., his measure of how good it is).

$$df = n - 1 = 14 \qquad 99\% \text{ confidence: } t = 2.977$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 69.53 \pm 2.977\,\frac{19.3}{\sqrt{15}} = 69.53 \pm 14.84$$

$$(54.69, 84.37)$$

$$95\% \text{ confidence: } t = 2.145$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 69.53 \pm 2.145\,\frac{19.3}{\sqrt{15}} = 69.53 \pm 10.69$$

$$(58.84, 80.22)$$

$$90\%\text{ confidence: } t = 1.761$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 69.53 \pm 1.761\,\frac{19.3}{\sqrt{15}} = 69.53 \pm 8.78$$

$$(60.75, 78.31)$$

(b) Does he have evidence at the 10%, 5%, 1% levels that his new routine is better?

$$H_0: \mu = 63.2$$

$$H_a: \mu > 63.2$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{69.53 - 63.2}{\frac{19.3}{\sqrt{15}}} = 1.270$$

$$1\text{ tail:}\quad 1.345 > 1.270 > 1.200$$

$$\text{p} - \text{value:}\quad 0.10 < P < 0.125 \qquad \text{Thus:}$$

$$P \nless .10 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .05 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

$$P \nless .01 \qquad \text{do not reject } H_0, \text{no evidence of } H_a$$

6. A simple random sample of 33 brown M&M's reveals that their average weight is 0.9128 grams with a standard deviation of 0.0395 grams. (a) Find 80%, 90%, 95%, 99% confidence intervals for the true average weight of (all) brown M&M's.

$$df = n - 1 = 32 \qquad 80\%\text{ confidence: } t = 1.309$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 0.9128 \pm 1.309\,\frac{0.0395}{\sqrt{33}} = 0.9128 \pm 0.0090$$

$$(0.9038, 0.9218)$$

$$90\%\text{ confidence: } t = 1.694$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 0.9128 \pm 1.309\,\frac{0.0395}{\sqrt{33}} = 0.9128 \pm 0.0116$$

$$(0.9012, 0.9244)$$

$$95\%\text{ confidence: } t = 2.037$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 0.9128 \pm 2.037\,\frac{0.0395}{\sqrt{33}} = 0.9128 \pm 0.0140$$

$$(0.8988, 0.9268)$$

$$99\%\text{ confidence: } t = 2.738$$

$$\bar{x} \pm t\,\frac{s}{\sqrt{n}} = 0.9128 \pm 2.738\,\frac{0.0395}{\sqrt{33}} = 0.9128 \pm 0.0188$$

$$(0.8940, 0.9316)$$

(b) Horatio, a lover of brown M&M's with too much time on his hands, grumbles that he believes that on average, a brown M&M weighs only 0.9085 grams. Do we have evidence that Horatio is underestimating the average weight of brown M&M's?

$$H_0: \mu = 0.9085$$

$$H_a: \mu > 0.9085$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{0.9128 - 0.9085}{\frac{0.0395}{\sqrt{33}}} = 0.625$$

$$1\text{ tail:}\quad 0.682 > 0.625$$

$$\text{p} - \text{value:}\quad 0.25 < P \qquad \text{Thus:}$$

$P \nless .10$ do not reject H_0, no evidence of H_a

$P \nless .05$ do not reject H_0, no evidence of H_a

$P \nless .01$ do not reject H_0, no evidence of H_a

7. A simple random sample of 20 statistics students during a statistics exam gives an average pulse rate 74.4 with a standard deviation of 10. (a) Find 90%, 95%, 99% confidence intervals for the average pulse rate of all statistics students during an exam.

$$df = n - 1 = 19 \qquad 90\% \text{ confidence: } t = 1.729$$

$$\bar{x} \pm t \frac{s}{\sqrt{n}} = 74.4 \pm 1.729 \frac{10}{\sqrt{20}} = 74.4 \pm 3.87$$

$$(70.53, 78.27)$$

$$95\% \text{ confidence: } t = 2.093$$

$$\bar{x} \pm t \frac{s}{\sqrt{n}} = 74.4 \pm 2.093 \frac{10}{\sqrt{20}} = 74.4 \pm 4.68$$

$$(69.72, 79.08)$$

$$99\% \text{ confidence: } t = 2.861$$

$$\bar{x} \pm t \frac{s}{\sqrt{n}} = 74.4 \pm 2.861 \frac{10}{\sqrt{20}} = 74.4 \pm 6.40$$

$$(68.00, 80.80)$$

(b) If a good exercise regimen would give such students a pulse rate of 60, do we have evidence at the 10%, 5%, 1% levels that a statistics exam has a greater effect on pulse rate than such exercise?

$$H_0: \mu = 60$$

$$H_a: \mu > 60$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{74.4 - 60}{\frac{10}{\sqrt{20}}} = 6.440$$

$$1 \text{ tail:} \quad 3.883 > 6.440$$

$$\text{p} - \text{value: } P < 0.0005 \qquad \text{Thus:}$$

$$P < .10 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P < .05 \qquad \text{reject } H_0, \text{evidence of } H_a$$

$$P < .01 \qquad \text{reject } H_0, \text{evidence of } H_a$$