

Has Fiscal Policy Helped Stabilize the Postwar Economy

Background Calculations

John Bailey Jones
Department of Economics
State University of New York at Albany
jbjones@csc.albany.edu

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1. The Basic Model

The numeraire is final output Y_t , which is produced by a unit mass of price-taking firms.

The production function facing any individual firm is

$$Y_t = Z_t^{1/\mu} K_t^a L_t^{1-a} (\bar{Y}_t/N_t),$$

$$0 < a < 1,$$

$$0 < \mu < 1/a,$$

where K_t and L_t denote capital and labor, respectively. N_t denotes the exogenous and deterministic population, and Z_t is the exogenous and stochastic productivity parameter.

There are externalities to production; each firm's production shifts in \bar{Y}_t/N_t .

Since all firms are identical and take \bar{Y}_t/N_t as given, total output follows

$$\bar{Y}_t = Y_t = Z_t K_t^{a\mu} L_t^{(1-a)\mu} N_t^{1-\mu}, \quad (1)$$

and the rental rate and wage are

$$r_t = a \frac{Y_t}{K_t}, \quad (2)$$

$$w_t = (1 - a) \frac{Y_t}{L_t}, \quad (3)$$

The government's budget constraint is

$$g_t Y_t + h_t Y_t - \tau_{Kt} r_t K_t - \tau_{Lt} w_t L_t + \delta \tau_{Kt} K_t + B_{gt} \leq \frac{1}{1 + i_{gt+1}} B_{gt+1}, \quad (4)$$

$$\begin{aligned}
0 &\leq g_t \leq 1, \\
0 &\leq h_t \leq 1, \\
-1 &\leq \tau_{Kt} \leq \bar{\tau} < 1, \\
-1 &\leq \tau_{Lt} \leq \bar{\tau} < 1, \\
B_{g0} &\text{ given.}
\end{aligned}$$

Here g_t and h_t denote the fraction of output consumed by the government and transferred back to consumers, respectively, while τ_{Kt} and τ_{Lt} are the *average* tax rates on capital and labor, respectively. B_{gt+1} denotes government bonds, which are discounted at the rate i_{gt+1} .

The economy is populated by a single representative family. Normalizing each member's labor endowment to 1, the family's flow utility from consumption (C_t) and leisure ($N_t - L_t$), is

$$\begin{aligned}
U(C_t, L_t) &= N_t \left[\ln \left(\frac{C_t}{N_t} \right) + \frac{\chi_t}{1-\gamma} \left(1 - \frac{L_t}{N_t} \right)^{1-\gamma} \right], \\
\gamma &\geq 0, \\
\frac{1}{1-\gamma} (x)^{1-\gamma} &\equiv \ln(x), \gamma = 1.
\end{aligned} \tag{5}$$

Note that government spending does not affect the returns to consumption or leisure. The family sells labor and rents capital, the law of motion for which is

$$K_{t+1} \leq (1 - \delta_t) K_t + I_t, \tag{6}$$

with $\{\kappa_{t+1}\}$ a stationary martingale difference sequence. The family's budget constraint is

$$(1 - \tau_{Kt}^*) r_t K_t + (1 - \tau_{Lt}^*) w_t L_t + \delta_t \tau_{Kt} K_t + h_t^* \bar{Y}_t + B_{gt} \geq C_t + I_t + \frac{1}{1 + i_{gt+1}} B_{gt+1}, \tag{7}$$

where τ_{Kt}^* and τ_{Lt}^* are the *marginal* tax rates on capital and labor, respectively. The transfer rate h_t^* ties together the budgets of the family and the government:

$$(1 - a) \tau_{Lt}^* + \tau_{Kt}^* \left(a - \delta_t \frac{K_t}{Y_t} \right) - h_t^* = (1 - a) \tau_{Lt} + \tau_{Kt} \left(a - \delta_t \frac{K_t}{Y_t} \right) - h_t. \tag{8}$$

In addition, these policies must satisfy

$$-1 \leq h_t^* \leq 1,$$

$$-1 \leq \tau_{Kt}^* \leq \bar{\tau} < 1,$$

$$-1 \leq \tau_{Lt}^* \leq \bar{\tau} < 1.$$

The family thus solves

$$\max_{\{C_t, L_t, K_{t+1}, B_{gt+1}, I_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \tilde{\beta}^t N_t \left[\ln \left(\frac{C_t}{N_t} \right) + \frac{\chi_t}{1-\gamma} \left(1 - \frac{L_t}{N_t} \right)^{1-\gamma} \right] \right\}$$

s.t. (6); (7);

K_0, B_{g0} given;

$C_t, I_t \geq 0$;

$L_t \in [0, N_t]$.

$E_t \{ \cdot \}$ denotes expectations conditional on information at time t , and $\tilde{\beta} \in (0, 1)$ is the family's discount factor. The first order conditions for an interior solution are

$$\chi_t \left(1 - \frac{L_t}{N_t} \right)^{-\gamma} = \frac{N_t}{C_t} (1 - \tau_{Lt}^*) w_t, \quad (9)$$

$$\frac{1}{C_t} = \tilde{\beta} \frac{N_{t+1}}{N_t} E_t \left\{ \frac{1}{C_{t+1}} \left[(1 - \tau_{Kt+1}^*) (r_{t+1} - \delta_{t+1}) + 1 \right] \right\}, \quad (10)$$

$$\frac{1}{1 + i_{gt+1}} = \tilde{\beta} \frac{N_{t+1}}{N_t} E_t \left\{ \frac{C_t}{C_{t+1}} \right\}, \quad (11)$$

along with the transversality conditions:

$$\lim_{j \rightarrow \infty} E_t \tilde{\beta}^{t+j} \left\{ \frac{N_{t+j} K_{t+j+1}}{C_{t+j}} \right\} = 0,$$

$$\lim_{j \rightarrow \infty} E_t \tilde{\beta}^{t+j} \left\{ \frac{N_{t+j} B_{gt+j+1}}{C_{t+j}} \right\} = 0.$$

The resource constraint for this economy is

$$Y_t = C_t + I_t + g_t Y_t. \quad (12)$$

As a specialization, I assume that technology follows

$$Z_t = Z_0 G_Z^t \cdot \exp(z_t), \quad (13)$$

where $G_Z > 0$ is the deterministic growth rate of technology and $Z_0 > 0$. Likewise, I assume that the depreciation rate follows

$$\delta_t = \delta + d_t, \quad (14)$$

with $\delta \in [0, 1)$, and that the leisure preference parameter χ_t follows

$$\chi_t = \chi \cdot \exp(x_t). \quad (15)$$

with $\chi > 0$. I further assume that (z_t, d_t, x_t) follows

$$\begin{pmatrix} z_{t+1} \\ d_{t+1} \\ x_{t+1} \end{pmatrix} = \Phi_1 \begin{pmatrix} z_t \\ d_t \\ x_t \end{pmatrix} + \nu_{t+1}, \quad (16)$$

with $\{\nu_t\}$ a stationary martingale difference sequence and the eigenvalues of Φ_1 all lying inside the unit circle.

Now suppose that $N_t = N_0 N^t$, with $N, N_0 > 0$. Let “ $\tilde{\cdot}$ ” denote per capita quantities, so that $\tilde{X}_t = X_t/N_t$. Then combining equations (1) through (4) and (9) through (13) shows that the equilibrium of this theoretical economy—excluding the laws of motion for fiscal policies and terminal conditions—is given by:

$$\chi_t (1 - \tilde{L}_t)^{-\gamma} = \frac{1}{\tilde{C}_t} (1 - \tau_{Lt}^*) (1 - a) \frac{\tilde{Y}_t}{\tilde{L}_t}, \quad (17)$$

$$\frac{1}{\tilde{C}_t} = \tilde{\beta} E_t \left\{ \frac{1}{\tilde{C}_{t+1}} \left[(1 - \tau_{Kt+1}^*) \left(a \frac{\tilde{Y}_{t+1}}{\tilde{K}_{t+1}} - \delta_{t+1} \right) + 1 \right] \right\}, \quad (18)$$

$$N \tilde{K}_{t+1} = (1 - \delta_t) \tilde{K}_t + (1 - g_t) \tilde{Y}_t - \tilde{C}_t, \quad (19)$$

$$[g_t + h_t - a \tau_{Kt} - (1 - a) \tau_{Lt}] \frac{\tilde{Y}_t}{\tilde{B}_{gt}} + \delta_t \tau_{Kt} \frac{\tilde{K}_t}{\tilde{B}_{gt}} + 1 = \tilde{\beta} N E_t \left\{ \frac{\tilde{C}_t}{\tilde{C}_{t+1}} \right\} \frac{\tilde{B}_{gt+1}}{\tilde{B}_{gt}}, \quad (20)$$

$$\tilde{Y}_t = Z_0 G_Z^t \cdot \exp(z_t) \tilde{K}_t^{a\mu} \tilde{L}_t^{(1-a)\mu}. \quad (21)$$

$$(1 - a) \tau_{Lt}^* + \tau_{Kt}^* \left(a - \delta_t \frac{\tilde{K}_t}{\tilde{Y}_t} \right) - h_t^* = (1 - a) \tau_{Lt} + \tau_{Kt} \left(a - \delta_t \frac{\tilde{K}_t}{\tilde{Y}_t} \right) - h_t. \quad (22)$$

Along a balanced growth path, with government policies suitably fixed, \tilde{L}_t is constant, while \tilde{C}_t , \tilde{K}_t , \tilde{Y}_t and \tilde{B}_{gt} grow at the constant rate

$$G = G_Z^{1/(1-a\mu)}.$$

Dividing through by the appropriate growth rates renders stationary the system given by equations (17) - (21). While I will continue to express the policy variables in levels, I will transform the other quantities, with $e_t \equiv \ln \left(\tilde{E}_t / G_E^t \right)$. I can then rewrite (17) - (21) as

$$\ell_t - \gamma \ln(1 - \exp(\ell_t)) = \ln\left(\frac{1-a}{\chi}\right) + \ln(1 - \tau_{Lt}^*) + y_t - c_t - x_t, \quad (23)$$

$$E_t \{\Delta c_{t+1}\} = \beta + \zeta_{1t+1} + E_t \left\{ \ln \left[(1 - \tau_{Kt+1}^*) (a \exp(y_{t+1} - k_{t+1}) - \delta - d_{t+1}) + 1 \right] \right\}, \quad (24)$$

$$\beta \equiv \ln(\tilde{\beta}/G) < 0,$$

$$\Delta k_{t+1} \approx \frac{\tilde{K}_{t+1}/G^{t+1} - \tilde{K}_t/G^t}{\tilde{K}_t/G^t} = (1 - NG) \exp(\Delta k_{t+1}) - \delta - d_t + (1 - g_t) \exp(y_t - k_t) - \exp(c_t - k_t), \quad (25)$$

$$\Delta b_{gt+1} - E_t \{\Delta c_{t+1}\} + \beta + \ln(NG) - \zeta_{2t+1} = \ln(1 + \exp(y_t - b_{gt})) \cdot [g_t + h_t - a\tau_{Kt} + (\delta + d_t) \tau_{Kt} \exp(-[y_t - k_t]) - (1 - a) \tau_{Lt}], \quad (26)$$

$$y_t = \ln(Z_0) + z_t + a\mu k_t + (1 - a) \mu \ell_t, \quad (27)$$

$$h_t^* - h_t = (1 - a) (\tau_{Lt}^* - \tau_{Lt}) + (\tau_{Kt}^* - \tau_{Kt}) (a - (\delta + d_t) \exp(-[y_t - k_t])). \quad (28)$$

$\zeta_{1t+1} \geq 0, \zeta_{2t+1} \geq 0$ capture the effect of interchanging the $\ln(\cdot)$ and expectation operators.

Now let's solve for the steady state. I will take $\tau_L^*, \tau_K^*, g, d_g \equiv b_g - y, \tau_L, \tau_K, \beta, \delta, \chi, G, N, a, Z_0$ as given, and solve for c, y, k, h, h^* , and ℓ . First, equations (24) and (25) yield

$$y - k = \lambda_4 \equiv \ln\left(\frac{1 - \exp(\beta) [1 - \delta (1 - \tau_K^*)]}{\exp(\beta) a (1 - \tau_K^*)}\right), \quad (29)$$

$$c - k = \lambda_5 \equiv \ln(\exp(\lambda_4) (1 - g) + 1 - \delta - NG). \quad (30)$$

Combining this with equations (23), (27), (26) and (28) gives us

$$\ell = \ln\left[\frac{(1 - \tau_L^*) (1 - a)}{\chi}\right] + \lambda_4 - \lambda_5 + \gamma \ln(1 - \exp(\ell)),$$

the fixed point of which is straightforward to find. I also get

$$k = \frac{\ln(Z_0) + (1 - a) \mu \ell - \lambda_4}{1 - a\mu},$$

$$h = (NG \exp(\beta) - 1) \exp(d_g) + [a\tau_K + (1 - a) \tau_L - \delta \tau_K \exp(-\lambda_4)] - g,$$

$$h^* = (1 - a) \tau_L^* + \tau_K^* (a - \delta \exp(-\lambda_4)) + (NG \exp(\beta) - 1) \exp(d_g) - g.$$

I then linearize (23) - (28) around these steady state values. Let “ $\hat{\cdot}$ ” denote deviations from the steady state, so that $\hat{x}_t = x_t - x$. First, equations (23) and (27) become

$$\lambda_0 \widehat{\ell}_t = \widehat{y}_t - \widehat{c}_t - \left[\frac{1}{1 - \tau_L^*} \right] \widehat{\tau}_{Lt}^* - x_t, \quad (31)$$

$$\lambda_0 \equiv \left[1 + \frac{\gamma}{1 - \exp(\ell)} \exp(\ell) \right], \quad (32)$$

$$\widehat{y}_t = (\lambda_1 + 1) \widehat{k}_t + \lambda_2 \widehat{c}_t + \frac{\lambda_2}{1 - \tau_L^*} \widehat{\tau}_{Lt}^* + \lambda_3 z_t + \lambda_2 x_t, \quad (33)$$

$$\lambda_1 \equiv \frac{a\mu}{1 - (1 - a)\mu/\lambda_0} - 1,$$

$$\lambda_2 \equiv -\frac{(1 - a)\mu/\lambda_0}{1 - (1 - a)\mu/\lambda_0},$$

$$\lambda_3 \equiv \frac{1}{1 - (1 - a)\mu/\lambda_0} = 1 - \lambda_2.$$

Applying equation (33), equations (24), (25) and (26) become

$$\begin{aligned} NG\Delta k_{t+1} = & -\exp(\lambda_4) \widehat{g}_t - \exp(\lambda_5) (\widehat{c}_t - \widehat{k}_t) - d_t \\ & + (1 - g) \exp(\lambda_4) \left[\lambda_1 \widehat{k}_t + \lambda_2 \widehat{c}_t + \frac{\lambda_2}{1 - \tau_L^*} \widehat{\tau}_{Lt}^* + \lambda_3 z_t + \lambda_2 x_t \right], \end{aligned} \quad (34)$$

$$\begin{aligned} E_t \{ \Delta \widehat{c}_{t+1} \} = & \left[\frac{\delta - a \exp(\lambda_4)}{\exp(-\beta)} \right] E_t \{ \widehat{\tau}_{Kt+1}^* \} - \frac{(1 - \tau_K^*)}{\exp(-\beta)} d_{t+1} \\ & + \lambda_7 E_t \left\{ \lambda_1 \widehat{k}_{t+1} + \lambda_2 \widehat{c}_{t+1} + \frac{\lambda_2}{1 - \tau_L^*} \widehat{\tau}_{Lt+1}^* + \lambda_3 z_{t+1} + \lambda_2 x_{t+1} \right\}, \end{aligned} \quad (35)$$

$$\lambda_7 \equiv \frac{a(1 - \tau_K^*)}{\exp(-\beta)} \exp(\lambda_4),$$

$$\begin{aligned} \Delta \widehat{b}_{gt+1} - E_t \{ \Delta \widehat{c}_{t+1} \} = & \lambda_8 \left[(\widehat{g}_t + \widehat{h}_t - a\widehat{\tau}_{Kt} - (1 - a)\widehat{\tau}_{Lt}) \right] \\ & + \lambda_8 \exp(-\lambda_4) \delta \left[\widehat{\tau}_{Kt} + \tau_K (\widehat{k}_t - \widehat{b}_{gt}) + \tau_K d_t / \delta \right] \\ & + \lambda_8 \lambda_9 \left[(\lambda_1 + 1) \widehat{k}_t + \lambda_2 \widehat{c}_t + \frac{\lambda_2}{1 - \tau_L^*} \widehat{\tau}_{Lt}^* + \lambda_3 z_t - \widehat{b}_{gt} + \lambda_2 x_{t+1} \right], \end{aligned} \quad (36)$$

$$\lambda_8 \equiv \frac{\exp(-d_g)}{\exp(\beta) NG},$$

$$\lambda_9 \equiv g + h - a\tau_K - (1 - a)\tau_L.$$

Now let's assume that marginal and average tax rates differ only by a constant, so that

$$\widehat{\tau}_{Kt}^* = \widehat{\tau}_{Kt}, \quad (37)$$

$$\widehat{\tau}_{Lt}^* = \widehat{\tau}_{Lt}. \quad (38)$$

Then one can write equations (34) (35), (36) and (14) as

$$\mathbf{S}_1 E_t \{ \mathbf{x}_{t+1} \} = \mathbf{S}_2 \mathbf{x}_t, \quad (39)$$

$$\mathbf{x}_t \equiv \left[\hat{c}_t \quad \hat{\tau}_{Kt} \quad \hat{\tau}_{Lt} \quad \hat{g}_t \quad \hat{h}_t \quad \hat{k}_t \quad \hat{b}_{gt} \quad z_t \quad d_t \quad x_t \right]^T,$$

where (note the transposition)

$$\mathbf{S}_1^T \equiv \begin{bmatrix} 0 & 1 - \lambda_7 \lambda_2 & -1 & 0 & 0 & 0 \\ 0 & [a \exp(\lambda_4) - \delta] \exp(\beta) & 0 & 0 & 0 & 0 \\ 0 & -\lambda_7 \lambda_2 / (1 - \tau_L^*) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ NG & -\lambda_7 \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\lambda_7 \lambda_3 & 0 & 1 & 0 & 0 \\ 0 & (1 - \tau_K^*) \exp(\beta) & 0 & 0 & 1 & 0 \\ 0 & -\lambda_7 \lambda_2 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{S}_2^T \equiv \begin{bmatrix} \left(\begin{array}{c} (1-g) \exp(\lambda_4) \lambda_2 \\ -\exp(\lambda_5) \end{array} \right) & 1 & \lambda_8 \lambda_9 \lambda_2 - 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda_8 [\exp(-\lambda_4) \delta - a] & 0 & 0 & 0 \\ \frac{(1-g) \exp(\lambda_4) \lambda_2}{1 - \tau_L^*} & 0 & \lambda_8 \left(\frac{\lambda_9 \lambda_2}{(1 - \tau_L^*)} - (1 - a) \right) & 0 & 0 & 0 \\ -\exp(\lambda_4) & 0 & \lambda_8 & 0 & 0 & 0 \\ 0 & 0 & \lambda_8 & 0 & 0 & 0 \\ \left(\begin{array}{c} NG + \exp(\lambda_5) \\ + (1-g) \exp(\lambda_4) \lambda_1 \end{array} \right) & 0 & \left(\begin{array}{c} \lambda_8 [\exp(-\lambda_4) \delta \tau_K \\ + \lambda_9 (\lambda_1 + 1)] \end{array} \right) & 0 & 0 & 0 \\ 0 & 0 & 1 - \lambda_8 [\exp(-\lambda_4) \delta \tau_K + \lambda_9] & 0 & 0 & 0 \\ (1-g) \exp(\lambda_4) \lambda_3 & 0 & \lambda_8 \lambda_9 \lambda_3 & \phi_{11} & \phi_{12} & \phi_{13} \\ -1 & 0 & \lambda_8 \exp(-\lambda_4) \tau_K & \phi_{21} & \phi_{22} & \phi_{23} \\ (1-g) \exp(\lambda_4) \lambda_2 & 0 & \lambda_8 \lambda_9 \lambda_2 & \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix},$$

Now let $i_t = \ln(I_t)$. It follows immediately from equation (12) that:

$$\exp(i_t) = (1 - g_t) \exp(y_t) - \exp(c_t). \quad (40)$$

In the steady state, this implies that

$$\exp(i - k) = (1 - g) \exp(\lambda_4) - \exp(\lambda_5),$$

so that, using equations (25), (29) and (30), we have

$$i - k = \lambda_6 \equiv \ln(NG - 1 + \delta).$$

And linearizing equation (40) yields

$$\exp(\lambda_6) \hat{i}_t = (1 - g) \exp(\lambda_4) \hat{y}_t - \exp(\lambda_4) \hat{g}_t - \exp(\lambda_5) \hat{c}_t. \quad (41)$$

Then, using (31), (33) and (41), one gets

$$\mathbf{w}_t = \mathbf{R}\mathbf{x}_t, \quad (42)$$

$$\mathbf{w}_t \equiv \begin{bmatrix} \widehat{y}_t & \widehat{c}_t & \widehat{i}_t & \widehat{\ell}_t & \widehat{y}_t - \widehat{\ell}_t & \widehat{k}_t & z_t & d_t & x_t & \widehat{\tau}_{Kt} & \widehat{\tau}_{Lt} & \widehat{g}_t & \widehat{h}_t \end{bmatrix}^{\mathbf{T}},$$

$$\mathbf{R} \equiv \begin{bmatrix} \lambda_2 & 0 & \frac{\lambda_2}{(1-\tau_L^*)} & 0 & 0 & \lambda_1 + 1 & 0 & \lambda_3 & 0 & \lambda_2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \left(\begin{array}{c} \lambda_{10}\lambda_2 \\ -\exp(\lambda_5 - \lambda_6) \end{array} \right) & 0 & \frac{\lambda_{10}\lambda_2}{1-\tau_L^*} & -\frac{\lambda_{10}}{1-g} & 0 & \lambda_{10}(\lambda_1 + 1) & 0 & \lambda_{10}\lambda_3 & 0 & \lambda_{10}\lambda_2 & 0 & 0 & 0 \\ -\lambda_3/\lambda_0 & 0 & \frac{-\lambda_3}{(1-\tau_L^*)} & 0 & 0 & (\lambda_1 + 1)/\lambda_0 & 0 & \lambda_3/\lambda_0 & 0 & -\lambda_3/\lambda_0 & 0 & 0 & 0 \\ \lambda_{11} & 0 & \frac{\lambda_{11}}{1-\tau_L^*} & 0 & 0 & \lambda_{12}(\lambda_1 + 1) & 0 & \lambda_{12}\lambda_3 & 0 & \lambda_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_{10} \equiv (1 - g) \exp(\lambda_4 - \lambda_6),$$

$$\lambda_{11} \equiv \lambda_2 + \lambda_3/\lambda_0,$$

$$\lambda_{12} \equiv 1 - 1/\lambda_0.$$

2. Revising the Model to Maximize Per Capita Utility

In many analyses, it is assumed that the household maximizes per capita, rather than total,

utility. In this case the household's problem becomes

$$\max_{\{C_t, L_t, K_{t+1}, B_{gt+1}, I_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \tilde{\beta}^t \left[\ln \left(\frac{C_t}{N_t} \right) + \frac{\chi_t}{1-\gamma} \left(1 - \frac{L_t}{N_t} \right)^{1-\gamma} \right] \right\}$$

$$s.t. \quad (6); (7);$$

$$K_0, B_{g0} \text{ given};$$

$$C_t, I_t \geq 0;$$

$$L_t \in [0, N_t].$$

The first order conditions for an interior solution are

$$\chi_t \left(1 - \frac{L_t}{N_t}\right)^{-\gamma} = \frac{N_t}{C_t} (1 - \tau_{Lt}^*) w_t, \quad (43)$$

$$\frac{1}{C_t} = \tilde{\beta} E_t \left\{ \frac{1}{C_{t+1}} \left[(1 - \tau_{Kt+1}^*) (r_{t+1} - \delta_{t+1}) + 1 \right] \right\}, \quad (44)$$

$$\frac{1}{1 + i_{gt+1}} = \tilde{\beta} E_t \left\{ \frac{C_t}{C_{t+1}} \right\}, \quad (45)$$

and the transversality conditions are:

$$\lim_{j \rightarrow \infty} E_t \tilde{\beta}^{t+j} \left\{ \frac{K_{t+j+1}}{C_{t+j}} \right\} = 0,$$

$$\lim_{j \rightarrow \infty} E_t \tilde{\beta}^{t+j} \left\{ \frac{B_{gt+j+1}}{C_{t+j}} \right\} = 0.$$

None of the other equations change.

Equations (18) and (20) then become

$$\frac{1}{\tilde{C}_t} = \frac{\tilde{\beta}}{N} E_t \left\{ \frac{1}{\tilde{C}_{t+1}} \left[(1 - \tau_{Kt+1}^*) \left(a \frac{\tilde{Y}_{t+1}}{\tilde{K}_{t+1}} - \delta_{t+1} \right) + 1 \right] \right\}, \quad (46)$$

$$[g_t + h_t - a\tau_{Kt} - (1 - a)\tau_{Lt}] \frac{\tilde{Y}_t}{\tilde{B}_{gt}} + \delta_t \tau_{Kt} \frac{\tilde{K}_t}{\tilde{B}_{gt}} + 1 = \tilde{\beta} E_t \left\{ \frac{\tilde{C}_t}{\tilde{C}_{t+1}} \right\} \frac{\tilde{B}_{gt+1}}{\tilde{B}_{gt}}, \quad (47)$$

which implies that equations (24) and (26) become

$$E_t \{ \Delta c_{t+1} \} = \beta + \zeta_{1t+1} + E_t \left\{ \ln \left[(1 - \tau_{Kt+1}^*) (a \exp(y_{t+1} - k_{t+1}) - \delta - d_{t+1}) + 1 \right] \right\}, \quad (48)$$

$$\beta \equiv \ln \left(\tilde{\beta} / [NG] \right) < 0,$$

$$\Delta b_{gt+1} - E_t \{ \Delta c_{t+1} \} + \beta + \ln(NG) - \zeta_{2t+1} = \ln(1 + \exp(y_t - b_{gt})) \cdot [g_t + h_t - a\tau_{Kt} + (\delta + d_t) \tau_{Kt} \exp(-[y_t - k_t]) - (1 - a)\tau_{Lt}]. \quad (49)$$

Comparing equations (48) and (49) to equations (24) and (26) shows that except for a redefinition of β , having the household maximize per capita rather than total utility has no effect on equations (23) through (42).

3. The Simplified Model with Fiscal Policy Feedback Rules: Version 1

To simplify the model further, I now assume that transfers adjust to keep the government's budget in balance every period. This means I can drop transfers, government debt and

the debt accumulation equation from our system. In addition, I can ignore the distinction between average and marginal tax rates, and focus solely on the latter. Among other things this implies that I can rewrite equation (39) as

$$\begin{aligned} \begin{bmatrix} \mathbf{S}_1 & \mathbf{H}_1 \\ \mathbf{0} & \mathbf{I}_2 \end{bmatrix} E_t \left\{ \begin{pmatrix} \mathbf{x}_{t+1} \\ \mathbf{z}_{t+1} \end{pmatrix} \right\} &= \begin{bmatrix} \mathbf{S}_2 & \mathbf{H}_2 \\ \mathbf{0} & \mathbf{\Phi}_1 \end{bmatrix} \mathbf{S}_2 \begin{pmatrix} \mathbf{x}_t \\ \mathbf{z}_t \end{pmatrix}, \\ \mathbf{x}_t &\equiv \begin{bmatrix} \widehat{c}_t & \widehat{\tau}_{Kt}^* & \widehat{\tau}_{Lt}^* & \widehat{g}_t & \widehat{k}_t \end{bmatrix}^{\mathbf{T}}, \\ \mathbf{z}_t &\equiv \begin{bmatrix} z_t & d_t & x_t \end{bmatrix}^{\mathbf{T}}, \end{aligned} \quad (50)$$

where

$$\begin{aligned} \mathbf{S}_1 &\equiv \begin{bmatrix} 0 & 0 & 0 & 0 & NG \\ 1 - \lambda_7 \lambda_2 & [a \exp(\lambda_4) - \delta] \exp(\beta) & -\frac{\lambda_7 \lambda_2}{1 - \tau_L^*} & 0 & -\lambda_7 \lambda_1 \end{bmatrix}, \\ \mathbf{H}_1 &\equiv \begin{bmatrix} 0 & 0 & 0 \\ -\lambda_7 \lambda_3 & (1 - \tau_K^*) \exp(\beta) & -\lambda_7 \lambda_2 \end{bmatrix}, \\ \mathbf{S}_2 &\equiv \begin{bmatrix} \begin{pmatrix} (1-g) \exp(\lambda_4) \lambda_2 \\ -\exp(\lambda_5) \end{pmatrix} & 0 & \frac{(1-g) \exp(\lambda_4) \lambda_2}{1 - \tau_L^*} & -\exp(\lambda_4) & \begin{pmatrix} NG + \exp(\lambda_5) \\ + (1-g) \exp(\lambda_4) \lambda_1 \end{pmatrix} \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \mathbf{H}_2 &\equiv \begin{bmatrix} (1-g) \exp(\lambda_4) \lambda_3 & -1 & (1-g) \exp(\lambda_4) \lambda_2 \\ 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

In a similar manner, equation (42) becomes

$$\begin{aligned} \mathbf{w}_t &= \begin{bmatrix} \mathbf{R}_0 & \mathbf{H}_0 \end{bmatrix} \begin{pmatrix} \mathbf{x}_t \\ \mathbf{z}_t \end{pmatrix}, \\ \mathbf{w}_t &\equiv \begin{bmatrix} \widehat{y}_t & \widehat{c}_t & \widehat{i}_t & \widehat{\ell}_t & \widehat{y}_t - \widehat{\ell}_t & \widehat{k}_t & \widehat{\tau}_{Kt}^* & \widehat{\tau}_{Lt}^* & \widehat{g}_t \end{bmatrix}^{\mathbf{T}}, \\ \mathbf{R}_0 &\equiv \begin{bmatrix} \lambda_2 & 0 & \frac{\lambda_2}{(1 - \tau_L^*)} & 0 & \lambda_1 + 1 \\ 1 & 0 & 0 & 0 & 0 \\ \begin{pmatrix} \lambda_{10} \lambda_2 \\ -\exp(\lambda_5 - \lambda_6) \end{pmatrix} & 0 & \frac{\lambda_{10} \lambda_2}{1 - \tau_L^*} & -\frac{\lambda_{10}}{1-g} & \lambda_{10} (\lambda_1 + 1) \\ -\lambda_3 / \lambda_0 & 0 & \frac{-\lambda_3}{(1 - \tau_L^*) \lambda_0} & 0 & (\lambda_1 + 1) / \lambda_0 \\ \lambda_{11} & 0 & \frac{\lambda_{11}}{1 - \tau_L^*} & 0 & \lambda_{12} (\lambda_1 + 1) \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \end{aligned} \quad (51)$$

$$\mathbf{H}_0 \equiv \begin{bmatrix} \lambda_3 & 0 & \lambda_2 \\ 0 & 0 & 0 \\ \lambda_{10}\lambda_3 & 0 & \lambda_{10}\lambda_2 \\ \lambda_3/\lambda_0 & 0 & -\lambda_3/\lambda_0 \\ \lambda_{12}\lambda_3 & 0 & \lambda_{11} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

But to complete the model, we must add a law of motion for taxes and government spending. In particular, I assume that fiscal policy is a function of its own lagged values and output:

$$\begin{aligned} \mathbf{v}_{t+1} &= -\tilde{\mathbf{F}}_0 \widehat{y}_{t+1} + \mathbf{F}_1 \begin{pmatrix} \widehat{y}_t \\ \mathbf{v}_t \end{pmatrix} + \mathbf{F}_2 \begin{pmatrix} \widehat{y}_{t-1} \\ \mathbf{v}_{t-1} \end{pmatrix} + \boldsymbol{\xi}_{t+1} + \boldsymbol{\psi}_{t+1}, \\ \mathbf{v}_{t+1} &\equiv \begin{pmatrix} \widehat{\tau}_{Kt+1}^* \\ \widehat{\tau}_{Lt+1}^* \\ \widehat{g}_{t+1} \end{pmatrix}. \end{aligned} \quad (52)$$

$\boldsymbol{\xi}_{t+1}$ and $\boldsymbol{\psi}_{t+1}$ are unobserved random variables. $\{\boldsymbol{\xi}_{t+1}\}$ is an exogenous forcing process that follows

$$\boldsymbol{\xi}_{t+1} = \boldsymbol{\Phi}_2 \boldsymbol{\xi}_t + \boldsymbol{\zeta}_{t+1}, \quad (53)$$

where $\{\boldsymbol{\zeta}_{t+1}\}$ is a martingale difference sequence and $\boldsymbol{\Phi}_2$ is a diagonal matrix whose elements all lie within the unit circle. In contrast, $\boldsymbol{\psi}_{t+1}$ is a forecast error,

$$\boldsymbol{\psi}_{t+1} \equiv \mathbf{v}_{t+1} - E_t \{\mathbf{v}_{t+1}\},$$

which is set endogenously. This allows me to rewrite (52) as

$$E_t \left\{ \begin{bmatrix} \tilde{\mathbf{F}}_0 & \mathbf{I}_3 \end{bmatrix} \begin{pmatrix} \widehat{y}_{t+1} \\ \mathbf{v}_{t+1} \end{pmatrix} - \boldsymbol{\xi}_{t+1} \right\} = \tilde{\mathbf{F}}_1 \begin{pmatrix} \widehat{y}_t \\ \mathbf{v}_t \end{pmatrix} + \mathbf{F}_2 \begin{pmatrix} \widehat{y}_{t-1} \\ \mathbf{v}_{t-1} \end{pmatrix}, \quad (54)$$

To simplify matters further, I set most of the elements of $\tilde{\mathbf{F}}_1$ and \mathbf{F}_2 to zero. In particular, I

assume that $\tilde{\mathbf{F}}_1$ can be written as

$$\tilde{\mathbf{F}}_1 = \begin{bmatrix} f_{11} & f_{14} & 0 & 0 \\ f_{12} & 0 & f_{15} & 0 \\ f_{13} & 0 & 0 & f_{16} \end{bmatrix},$$

and \mathbf{F}_2 can be written similarly.

At this point, the model is completely specified. Before the model can be put in its final

form, one must use the first row of equation (51) to rewrite the left hand side of equation (54) as

$$\begin{pmatrix} y_t \\ \mathbf{v}_t \end{pmatrix} = \mathbf{R}_1 \mathbf{x}_t + \mathbf{H}_3 \mathbf{z}_t, \quad (55)$$

$$\mathbf{R}_1 \equiv \begin{bmatrix} \lambda_2 & 0 & \frac{\lambda_2}{(1-\tau_L^*)} & 0 & \lambda_1 + 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{H}_3 \equiv \begin{bmatrix} \lambda_3 & 0 & \lambda_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} \tilde{\mathbf{F}}_0 & \mathbf{I}_3 \end{bmatrix} \begin{pmatrix} \hat{y}_{t+1} \\ \mathbf{v}_{t+1} \end{pmatrix} = \mathbf{F}_0 \mathbf{x}_{t+1} + \mathbf{H}_4 \mathbf{z}_{t+1}, \quad (56)$$

$$\mathbf{F}_0 \equiv \begin{bmatrix} \tilde{\mathbf{F}}_0 & \mathbf{I}_3 \end{bmatrix} \mathbf{R}_1,$$

$$\mathbf{H}_4 \equiv \begin{bmatrix} \tilde{\mathbf{F}}_0 & \mathbf{I}_3 \end{bmatrix} \mathbf{H}_3,$$

Similarly one gets

$$\tilde{\mathbf{F}}_1 \begin{pmatrix} \hat{y}_t \\ \mathbf{v}_t \end{pmatrix} = \mathbf{F}_1 \mathbf{x}_t + \mathbf{H}_5 \mathbf{z}_t, \quad (57)$$

$$\mathbf{F}_1 \equiv \tilde{\mathbf{F}}_1 \mathbf{R}_1,$$

$$\mathbf{H}_5 \equiv \tilde{\mathbf{F}}_1 \mathbf{H}_3.$$

Finally, combine equations (50) and (54) through (55) to get

$$\mathbf{A}_0 E_t \left\{ \begin{pmatrix} \mathbf{x}_{t+1} \\ \hat{y}_t \\ \mathbf{v}_t \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{pmatrix} \right\} = \mathbf{A}_1 \begin{pmatrix} \mathbf{x}_t \\ \hat{y}_{t-1} \\ \mathbf{v}_{t-1} \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{pmatrix}, \quad (58)$$

$$\mathbf{A}_0 \equiv \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} & \mathbf{H}_1 & \mathbf{0} \\ (2 \times 5) & (2 \times 4) & (2 \times 3) & (2 \times 3) \\ \mathbf{0} & \mathbf{I}_4 & \mathbf{0} & \mathbf{0} \\ (4 \times 5) & (4 \times 4) & (4 \times 3) & (4 \times 3) \\ \mathbf{F}_0 & \mathbf{0} & \mathbf{H}_4 & -\mathbf{I}_3 \\ (3 \times 5) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_3 & \mathbf{0} \\ (3 \times 5) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 \\ (3 \times 5) & (3 \times 4) & (3 \times 3) & (3 \times 3) \end{bmatrix},$$

$$\mathbf{A}_1 \equiv \begin{bmatrix} \mathbf{S}_2 & \mathbf{0} & \mathbf{H}_2 & \mathbf{0} \\ (2 \times 5) & (2 \times 4) & (2 \times 3) & (2 \times 3) \\ \mathbf{R}_1 & \mathbf{0} & \mathbf{H}_3 & \mathbf{0} \\ (4 \times 5) & (4 \times 4) & (4 \times 3) & (4 \times 3) \\ \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{H}_5 & \mathbf{0} \\ (3 \times 5) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{\Phi}_1 & \mathbf{0} \\ (3 \times 5) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Phi}_2 \\ (3 \times 5) & (3 \times 4) & (3 \times 3) & (3 \times 3) \end{bmatrix}.$$

The system is completed with a restriction on its innovation process:

$$\left\{ \left(\begin{array}{c} \widehat{k}_{t+1} \\ \widehat{y}_t \\ \mathbf{v}_t \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{array} \right) - E_t \left\{ \left(\begin{array}{c} \widehat{k}_{t+1} \\ \widehat{y}_t \\ \mathbf{v}_t \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{array} \right) \right\} \right\} = \left\{ \left(\begin{array}{c} \mathbf{0} \\ (5 \times 1) \\ \boldsymbol{\nu}_{t+1} \\ \boldsymbol{\zeta}_{t+1} \end{array} \right) \right\}. \quad (59)$$

And, recalling equation (51), the variables of most interest can be recovered with

$$\left(\begin{array}{c} \mathbf{w}_t \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{array} \right) = \begin{bmatrix} \mathbf{R}_0 & \mathbf{0} & \mathbf{H}_0 & \mathbf{0} \\ (9 \times 5) & (9 \times 4) & (9 \times 3) & (9 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_3 & \mathbf{0} \\ (3 \times 5) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 \\ (3 \times 5) & (3 \times 4) & (3 \times 3) & (3 \times 3) \end{bmatrix} \left(\begin{array}{c} \mathbf{x}_t \\ \widehat{y}_{t-1} \\ \mathbf{v}_{t-1} \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{array} \right). \quad (60)$$

4. The Simplified Model with Fiscal Policy Feedback Rules: Version 2

Let's consider a variant of the model previously discussed. There are two differences. The first is that fiscal policy is no a function of employment, rather than output. The second is that government spending depends of 4 lags of spending and employment rather than 2. (The tax rates still depend on two lags.)

The first effect of these changes is that equation (54) becomes

$$E_t \left\{ \left[\widetilde{\mathbf{F}}_0 \quad \mathbf{I}_3 \right] \left(\begin{array}{c} \widehat{\ell}_{t+1} \\ \mathbf{v}_{t+1} \end{array} \right) - \boldsymbol{\xi}_{t+1} \right\} = \widetilde{\mathbf{F}}_1 \left(\begin{array}{c} \widehat{\ell}_t \\ \mathbf{v}_t \end{array} \right) + \mathbf{F}_2 \left(\begin{array}{c} \widehat{\ell}_{t-1} \\ \mathbf{v}_{t-1} \end{array} \right) + \quad (61)$$

$$\mathbf{F}_3 \begin{pmatrix} \widehat{\ell}_{t-2} \\ \mathbf{v}_{t-2} \end{pmatrix} + \mathbf{F}_4 \begin{pmatrix} \widehat{\ell}_{t-3} \\ \mathbf{v}_{t-3} \end{pmatrix}.$$

Note that the first 2 rows of \mathbf{F}_3 and \mathbf{F}_4 consist of zeros.

In addition, equation (55) becomes

$$\begin{pmatrix} \widehat{\ell}_t \\ \mathbf{v}_t \end{pmatrix} = \mathbf{R}_1 \mathbf{x}_t + \mathbf{H}_3 \mathbf{z}_t, \quad (62)$$

$$\mathbf{R}_1 \equiv \begin{bmatrix} -\lambda_3/\lambda_0 & 0 & \frac{-\lambda_3}{(1-\tau_L^*)\lambda_0} & 0 & (\lambda_1 + 1)/\lambda_0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{H}_3 \equiv \begin{bmatrix} \lambda_3/\lambda_0 & 0 & -\lambda_3/\lambda_0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

so that

$$\mathbf{A}_0 E_t \left\{ \begin{pmatrix} \mathbf{x}_{t+1} \\ \widehat{\ell}_t \\ \mathbf{v}_t \\ \widehat{\ell}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{\ell}_{t-2} \\ \mathbf{v}_{t-2} \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{pmatrix} \right\} = \mathbf{A}_1 \left\{ \begin{pmatrix} \mathbf{x}_t \\ \widehat{\ell}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{\ell}_{t-2} \\ \mathbf{v}_{t-2} \\ \widehat{\ell}_{t-3} \\ \mathbf{v}_{t-3} \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{pmatrix} \right\}, \quad (63)$$

$$\mathbf{A}_0 \equiv \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_1 & \mathbf{0} \\ (2 \times 5) & (2 \times 4) & (2 \times 4) & (2 \times 4) & (2 \times 3) & (2 \times 3) \\ \mathbf{0} & \mathbf{I}_4 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (4 \times 5) & (4 \times 4) & (4 \times 4) & (4 \times 4) & (4 \times 3) & (4 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_4 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (4 \times 5) & (4 \times 4) & (4 \times 4) & (4 \times 4) & (4 \times 3) & (4 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_4 & \mathbf{0} & \mathbf{0} \\ (4 \times 5) & (4 \times 4) & (4 \times 4) & (4 \times 4) & (4 \times 3) & (4 \times 3) \\ \mathbf{F}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_4 & -\mathbf{I}_3 \\ (3 \times 5) & (3 \times 4) & (3 \times 4) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 & \mathbf{0} \\ (3 \times 5) & (3 \times 4) & (3 \times 4) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 \\ (3 \times 5) & (3 \times 4) & (3 \times 4) & (3 \times 4) & (3 \times 3) & (3 \times 3) \end{bmatrix},$$

$$\mathbf{A}_1 \equiv \begin{bmatrix} \mathbf{S}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_2 & \mathbf{0} \\ (2 \times 5) & (2 \times 4) & (2 \times 4) & (2 \times 4) & (2 \times 3) & (2 \times 3) \\ \mathbf{R}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_3 & \mathbf{0} \\ (4 \times 5) & (4 \times 4) & (4 \times 4) & (4 \times 4) & (4 \times 3) & (4 \times 3) \\ \mathbf{0} & \mathbf{I}_4 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (4 \times 5) & & (4 \times 4) & (4 \times 4) & (4 \times 3) & (4 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_4 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (4 \times 5) & (4 \times 4) & & (4 \times 4) & (4 \times 3) & (4 \times 3) \\ \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 & \mathbf{F}_4 & \mathbf{H}_5 & \mathbf{0} \\ (3 \times 5) & (3 \times 4) & (3 \times 4) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Phi}_1 & \mathbf{0} \\ (3 \times 5) & (3 \times 4) & (3 \times 4) & (3 \times 4) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Phi}_2 \\ (3 \times 5) & (3 \times 4) & (3 \times 4) & (3 \times 4) & (3 \times 3) & (3 \times 3) \end{bmatrix},$$

with \mathbf{F}_0 , \mathbf{H}_4 , \mathbf{F}_1 and \mathbf{H}_5 computed as before, but with the revised values of \mathbf{R}_0 and \mathbf{H}_3 .

The system is completed by restricting its innovation process:

$$\left\{ \begin{pmatrix} \widehat{k}_{t+1} \\ \widehat{l}_t \\ \mathbf{v}_t \\ \widehat{l}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{l}_{t-2} \\ \mathbf{v}_{t-2} \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{pmatrix} - E_t \left\{ \begin{pmatrix} \widehat{k}_{t+1} \\ \widehat{l}_t \\ \mathbf{v}_t \\ \widehat{l}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{l}_{t-2} \\ \mathbf{v}_{t-2} \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{pmatrix} \right\} \right\} = \left\{ \begin{pmatrix} \mathbf{0} \\ (13 \times 1) \\ \boldsymbol{\nu}_{t+1} \\ \boldsymbol{\zeta}_{t+1} \end{pmatrix} \right\}. \quad (64)$$

And, recalling equation (51), the variables of most interest can be recovered with

$$\begin{pmatrix} \mathbf{w}_t \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{pmatrix} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{0} & \mathbf{H}_0 & \mathbf{0} \\ (9 \times 5) & (9 \times 12) & (9 \times 3) & (9 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_3 & \mathbf{0} \\ (3 \times 5) & (3 \times 12) & & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 \\ (3 \times 5) & (3 \times 12) & (3 \times 3) & \end{bmatrix} \begin{pmatrix} \mathbf{x}_t \\ \widehat{l}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{l}_{t-2} \\ \mathbf{v}_{t-2} \\ \widehat{l}_{t-3} \\ \mathbf{v}_{t-3} \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{pmatrix}. \quad (65)$$

5. The Simplified Model with Fiscal Policy Feedback Rules: Version 3

Let's consider a yet another variant of the models previously discussed. This version is the same as version 2, except that labor taxes depend on both employment and output.

The first effect of these changes is that equation (54) becomes

$$E_t \left\{ \begin{bmatrix} \tilde{\mathbf{F}}_0 & \mathbf{I}_3 \end{bmatrix} \begin{pmatrix} \widehat{y}_{t+1} \\ \widehat{\ell}_{t+1} \\ \mathbf{v}_{t+1} \end{pmatrix} - \boldsymbol{\xi}_{t+1} \right\} = \tilde{\mathbf{F}}_1 \begin{pmatrix} \widehat{y}_t \\ \widehat{\ell}_t \\ \mathbf{v}_t \end{pmatrix} + \mathbf{F}_2 \begin{pmatrix} \widehat{y}_{t-1} \\ \widehat{\ell}_{t-1} \\ \mathbf{v}_{t-1} \end{pmatrix} + \mathbf{F}_3 \begin{pmatrix} \widehat{y}_{t-2} \\ \widehat{\ell}_{t-2} \\ \mathbf{v}_{t-2} \end{pmatrix} + \mathbf{F}_4 \begin{pmatrix} \widehat{y}_{t-3} \\ \widehat{\ell}_{t-3} \\ \mathbf{v}_{t-3} \end{pmatrix}. \quad (66)$$

Note that the first 2 rows of \mathbf{F}_3 and \mathbf{F}_4 consist of zeros.

In addition, equation (55) becomes

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{\ell}_t \\ \mathbf{v}_t \end{pmatrix} = \mathbf{R}_1 \mathbf{x}_t + \mathbf{H}_3 \mathbf{z}_t, \quad (67)$$

$$\mathbf{R}_1 \equiv \begin{bmatrix} \lambda_2 & 0 & \frac{\lambda_2}{(1-\tau_L^*)\lambda_0} & 0 & \lambda_1 + 1 \\ -\lambda_3/\lambda_0 & 0 & \frac{-\lambda_3}{(1-\tau_L^*)\lambda_0} & 0 & (\lambda_1 + 1)/\lambda_0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{H}_3 \equiv \begin{bmatrix} \lambda_3 & 0 & \lambda_2 \\ \lambda_3/\lambda_0 & 0 & -\lambda_3/\lambda_0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

so that

$$\mathbf{A}_0 E_t \left\{ \begin{pmatrix} \mathbf{x}_{t+1} \\ \widehat{y}_t \\ \widehat{\ell}_t \\ \mathbf{v}_t \\ \widehat{y}_{t-1} \\ \widehat{\ell}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{y}_{t-2} \\ \widehat{\ell}_{t-2} \\ \mathbf{v}_{t-2} \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{pmatrix} \right\} = \mathbf{A}_1 \begin{pmatrix} \mathbf{x}_t \\ \widehat{y}_{t-1} \\ \widehat{\ell}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{y}_{t-2} \\ \widehat{\ell}_{t-2} \\ \mathbf{v}_{t-2} \\ \widehat{y}_{t-3} \\ \widehat{\ell}_{t-3} \\ \mathbf{v}_{t-3} \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{pmatrix}, \quad (68)$$

$$\mathbf{A}_0 \equiv \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_1 & \mathbf{0} \\ (2 \times 5) & (2 \times 5) & (2 \times 5) & (2 \times 5) & (2 \times 3) & (2 \times 3) \\ \mathbf{0} & \mathbf{I}_5 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 3) & (5 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_5 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 3) & (5 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_5 & \mathbf{0} & \mathbf{0} \\ (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 3) & (5 \times 3) \\ \mathbf{F}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_4 & -\mathbf{I}_3 \\ (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 & \mathbf{0} \\ (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 \\ (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 3) & (3 \times 3) \end{bmatrix},$$

$$\mathbf{A}_1 \equiv \begin{bmatrix} \mathbf{S}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_2 & \mathbf{0} \\ (2 \times 5) & (2 \times 5) & (2 \times 5) & (2 \times 5) & (2 \times 3) & (2 \times 3) \\ \mathbf{R}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_3 & \mathbf{0} \\ (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 3) & (5 \times 3) \\ \mathbf{0} & \mathbf{I}_5 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 3) & (5 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_5 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 5) & (5 \times 3) & (5 \times 3) \\ \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 & \mathbf{F}_4 & \mathbf{H}_5 & \mathbf{0} \\ (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Phi}_1 & \mathbf{0} \\ (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Phi}_2 \\ (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 5) & (3 \times 3) & (3 \times 3) \end{bmatrix},$$

with \mathbf{F}_0 , \mathbf{H}_4 , \mathbf{F}_1 and \mathbf{H}_5 computed as before, but with the revised values of \mathbf{R}_0 and \mathbf{H}_3 .

The system is completed by restricting its innovation process:

$$\left\{ \begin{pmatrix} \widehat{k}_{t+1} \\ \widehat{y}_t \\ \widehat{\ell}_t \\ \mathbf{v}_t \\ \widehat{y}_{t-1} \\ \widehat{\ell}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{y}_{t-2} \\ \widehat{\ell}_{t-2} \\ \widehat{\mathbf{v}}_{t-2} \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{pmatrix} - E_t \left\{ \begin{pmatrix} \widehat{k}_{t+1} \\ \widehat{y}_t \\ \widehat{\ell}_t \\ \mathbf{v}_t \\ \widehat{y}_{t-1} \\ \widehat{\ell}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{y}_{t-2} \\ \widehat{\ell}_{t-2} \\ \mathbf{v}_{t-2} \\ \mathbf{z}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{pmatrix} \right\} = \left\{ \begin{pmatrix} \mathbf{0} \\ (16 \times 1) \\ \boldsymbol{\nu}_{t+1} \\ \boldsymbol{\zeta}_{t+1} \end{pmatrix} \right\}. \quad (69)$$

And, recalling equation (51), the variables of most interest can be recovered with

$$\begin{pmatrix} \mathbf{w}_t \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{pmatrix} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{0} & \mathbf{H}_0 & \mathbf{0} \\ (9 \times 5) & (9 \times 15) & (9 \times 3) & (9 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_3 & \mathbf{0} \\ (3 \times 5) & (3 \times 15) & (3 \times 3) & (3 \times 3) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 \\ (3 \times 5) & (3 \times 15) & (3 \times 3) & \end{bmatrix} \begin{pmatrix} \mathbf{x}_t \\ \widehat{\mathbf{y}}_{t-1} \\ \widehat{\mathbf{l}}_{t-1} \\ \mathbf{v}_{t-1} \\ \widehat{\mathbf{y}}_{t-2} \\ \widehat{\mathbf{l}}_{t-2} \\ \mathbf{v}_{t-2} \\ \widehat{\mathbf{y}}_{t-3} \\ \widehat{\mathbf{l}}_{t-3} \\ \mathbf{v}_{t-3} \\ \mathbf{z}_t \\ \boldsymbol{\xi}_t \end{pmatrix}. \tag{70}$$