Economics 701: Macroeconomics II
Spring 2008
Professor Jones

Midterm Examination
Answer Key

1. The three main equations of our ad hoc model are:

$$\begin{aligned} y_t &= p_t - p_t^e, & \text{(AS)} \\ m_t - p_t &= y_t - i_t, & \text{(LM)} \\ y_t &= -\left[i_t - \left(p_{t+1}^e - p_t\right)\right] = p_{t+1}^e - p_t - i_t. & \text{(IS)} \end{aligned}$$

(a) Combining the (IS) and (LM) equations produces

$$p_{t+1}^e - p_t - i_t = m_t - p_t + i_t,$$

which reduces to

$$m_t = -2i_t + p_{t+1}^e. \quad \text{(MD)}$$

Given that the nominal interest rate i_t is the opportunity cost of holding money, it is not surprising that we see a negative relationship between i_t and m_t: all else equal higher nominal interest rates imply lower money holdings. A more precise interpretation (provided by several students) is that increasing the nominal interest rate requires the monetary authority to reduce the nominal money supply: reducing m_t shifts the LM curve up and to the left, leading to higher interest rates in an IS-LM equilibrium.

(b) Now suppose that the monetary authority sets the nominal interest rate using the following variant of the "Taylor Rule":

$$i_t = \alpha y_t + \beta p_t, \quad \alpha, \beta \geq 0. \quad \text{(TR)}$$

Combining equations (MD) and (TR) produces

$$m_t = -2\alpha y_t - 2\beta p_t + p_{t+1}^e. \quad \text{(MS)}$$

The interpretation of this equation is straightforward. Output and price levels of 0 are treated as ideal. (If one wants a justification for a lower bound on prices, it is worth noting that price deflation is often viewed as a damaging event.) If output and/or prices fall below their optimal level, the money supply is raised. This shifts the aggregate demand (AD) curve to the right, increasing both prices and output. If output and prices get too high, the money supply is lowered, shifting the AD curve left and decreasing both prices and output.

Expected prices enter with a positive coefficient because of their effect on the IS curve. As p_{t+1}^e increases, the real interest rate associated with any nominal interest rate falls. The IS curve then shifts to the right, raising the equilibrium value of i_t in the IS-LM model. The monetary authority offsets this by shifting the LM curve down and to the right, which requires an increase in m_t.

(c) Combining the (IS) and (LM) curves produces

$$m_t - p_t - y_t = y_t - p^e_{t+1} + p_t,$$

which reduces to

$$y_t = \frac{1}{2}m_t + \frac{1}{2}p^e_{t+1} - p_t. \tag{AD}$$

Insering equation (MS) produces

$$\begin{aligned} y_t &= -\alpha y_t - \beta p_t + \frac{1}{2}p^e_{t+1} + \frac{1}{2}p^e_{t+1} - p_t \\ &= -\frac{1+\beta}{1+\alpha}p_t + \frac{1}{1+\alpha}p^e_{t+1}. \end{aligned} \tag{AD$'$}$$

Finally, combing the aggregate demand and aggregate supply curves produces

$$p_t - p^e_t = -\frac{1+\beta}{1+\alpha}p_t + \frac{1}{1+\alpha}p^e_{t+1},$$

or

$$p_t = \frac{1+\alpha}{2+\alpha+\beta}p^e_t + \frac{1}{2+\alpha+\beta}p^e_{t+1}. \tag{EQP}$$

(d) Applying the law of iterated expectations to equation (EQP) shows that

$$E_{t-1}(p_t) = \frac{1+\alpha}{2+\alpha+\beta}E_{t-1}(E_{t-1}(p_t)) + \frac{1}{2+\alpha+\beta}E_{t-1}(E_t(p_{t+1})),$$

or

$$E_{t-1}(p_t) = \frac{1}{1+\beta}E_{t-1}(p_{t+1}).$$

Rewriting this equation using lag operators, we get

$$E_{t-1}\left(\left(1 - \frac{1}{1+\beta}L^{-1}\right)p_t\right) = -E_{t-1}\left(\left(1-(1+\beta)L\right)p_{t+1}\right) = 0.$$

With $\beta > 0$, the only solution that does not lead to exploding prices is

$$E_{t-1}(p_t) = E_{t-1}\left(\frac{1}{1-\frac{1}{1+\beta}L^{-1}}0\right) = 0.$$

But then it immediately follows from equation (EQP) that $p_t = 0$ as well.

(e) With $p_t = p^e_t = 0$, it follows immediately from equation (AS) that $y_t = 0$ as well. Output is at its full employment level and prices equal zero. Given that agents have rational expectations, the government has no informational advantages, and there are no external shocks to the economy, it is not surprising that the economy is at full employment. Prices are pinned down by the Taylor rule. Were prices to rise, the monetary authority would increase interest rates by reducing the money supply, this shifts the AD curve left, immediately driving prices back down. If prices fall below zero, an interest rate cut would increase the money supply and reverse the fall.

2. We are considering a standard Lucas tree model with discrete shocks. The preferences of the representative consumer are

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\left[c_t^{1-\sigma}-1\right]\right),\quad 0<\beta<1,\quad \sigma>0.$$

The economy starts off with each consumer owning one dividend-producing tree. Dividends can take on two values, d_L and $d_H > d_L$, and follow a symmetric two-state Markov process:

$$\begin{aligned} f(d_L,d_L) &= \Pr\left(d_{t+1}=d_L\,|\,d_t=d_L\right)=\pi=f(d_H,d_H),\\ f(d_H,d_L) &= \Pr\left(d_{t+1}=d_H\,|\,d_t=d_L\right)=1-\pi=f(d_L,d_H).\end{aligned}$$

(a) Let $q(d',d)$ denote the price of the one-step-ahead contingent claim that pays off when $d_{t+1}=d'$. Writing the consumer's problem as a Lagrangean, we get

$$\begin{aligned} &V(x_t,d_t)=\\ &\min_{\lambda_t\geq 0}\ \max_{c_t\geq 0,\ s_{t+1},\ z(d')}\ \frac{c_t^{1-\sigma}-1}{1-\sigma}+\lambda_t\left(x_t-c_t-p_ts_{t+1}-\sum_{d'\in\{d_L,d_H\}}q(d',d_t)\,z(d')\right)\\ &\quad+\ \beta\sum_{d_{t+1}\in\{d_L,d_H\}}f(d_{t+1},d_t)\times V\big(z(d_{t+1})+\left[p_{t+1}(d_{t+1})+d_{t+1}\right]s_{t+1},d_{t+1}\big),\end{aligned}$$

The FOC for an interior solution are:

$$\begin{aligned} c_t^{-\sigma}&=\lambda_t,\\ \lambda_tp_t&=\beta\sum_{d_{t+1}\in\{d_L,d_H\}}f(d_{t+1},d_t)\times\frac{\partial V\left(x_{t+1}(d'),d'\right)}{\partial x_{t+1}}\left[p_{t+1}(d_{t+1})+d_{t+1}\right],\\ \lambda_tq(d',d_t)&=\beta\frac{\partial V\left(x_{t+1}(d'),d'\right)}{\partial x_{t+1}}f(d',d_t),\quad d'\in\{d_L,d_H\}.\end{aligned}$$

Since (following Benveniste-Scheinkman),

$$\frac{\partial V[t]}{\partial x_t}=\lambda_t,$$

the Euler equations are

$$p_tc_t^{-\sigma}=\beta E_t\left(c_{t+1}(d_{t+1})^{-\sigma}\left[p_{t+1}(d_{t+1})+d_{t+1}\right]\right),\tag{EE1}$$

$$q(d',d_t)=\beta\left(\frac{c_t}{c_{t+1}(d')}\right)^{\sigma}f(d',d_t),\quad d'\in d'\in\{d_L,d_H\}.\tag{EE2}$$

(b) Given the random variable d_0, the conditional density $f(d_{t+1}|\,d_t)$, and the initial endowments $s_0=1$ and $z(d_0)=0$, a recursive rational expectations equilibrium consists of pricing functions $p(d)$ and $q(d',d)$, a value function $V(x,d)$, and decision functions $c(x,d)$, $s(x,d)$, and $z(d',x,d)$ such that:

1. Given the pricing functions $p(d)$ and $q(d', d)$, the value and policy functions $V(x, d)$, $c(x, d)$, $s(x, d)$, and $z(d', x, d)$ solve the consumer's problem.
2. Markets clear: for $x = p(d) + d$, $c(x, d) = d$, $s(x, d) = 1$, and $z(d', x, d) = 0$.

(c) To achieve equilibrium,we impose $s_{t+1} = 1$, $z(d_{t+1}) = 0$, $\forall d_{t+1}$ so that $c_t = d_t$. It follows from equation (EE2) that the equilibrium price for a contingent claim is

$$q(d', d_t) = \beta \left(\frac{d_t}{d'}\right)^{\sigma} f(d', d_t).$$

Continuing, we get

$$\begin{aligned} q_{LL} &\equiv q(d_L, d_L) = \beta\pi \\ q_{HL} &\equiv q(d_H, d_L) = \beta \left(\frac{d_L}{d_H}\right)^{\sigma} (1-\pi). \end{aligned}$$

(d) It follows from arbitrage arguments that if an asset pays $w(d)$ units of consumption goods when $d_{t+1} = d$, its price is

$$p_t^w = \sum_{d_{t+1} \in \{d_L, d_H\}} w(d_{t+1}) q(d_{t+1}, d_t).$$

When the asset is a risk-free discount bond, $w(d) = 1$. Imposing equation (EE2), it follows that the price of a risk-free bond, $R^{-1}(d_t)$ is given by

$$R^{-1}(d_t) = \beta \sum_{d_{t+1}} f(d_{t+1}, d_t) \left(\frac{d_t}{d_{+1}}\right)^{\sigma} = \beta d_t^{\sigma} E\left(d_{t+1}^{-\sigma} \middle| d_t\right).$$

Continuing, we have

$$R_L^{-1} \equiv R^{-1}(d_L) = \beta \left[\pi \left(\frac{d_L}{d_L}\right)^{\sigma} + (1-\pi)\left(\frac{d_L}{d_H}\right)^{\sigma}\right] = \beta \left[\pi + (1-\pi)\left(\frac{d_L}{d_H}\right)^{\sigma}\right].$$

(e) Now we consider the asset i_L, sold when *current* dividends are low. If dividends in period $t+1$ are low ($d_{t+1} = d_L$), one unit of asset i_L will deliver $1/\pi$ units of consumption goods; if dividends are high, the asset pays nothing.

1. Let p_{iL} denote the price of this asset. Using the pricing kernel,we find that

$$p_{iL} = \frac{1}{\pi} q_{LL} + 0 \cdot q_{HL} = \beta.$$

2. The expected rate of return on asset i_L is

$$R_L^i = \frac{1}{p_{iL}} \left[\pi \cdot \frac{1}{\pi} + (1-\pi) \cdot 0\right] = \beta^{-1}.$$

3. Using our answer from part (d) shows that when dividends are low

$$R_L = \beta^{-1} \left[\pi + (1-\pi)\left(\frac{d_L}{d_H}\right)^{\sigma}\right]^{-1} > \beta^{-1},$$

as $d_H > d_L$. We therefore see that the expected return on asset i_L is less than the expected return on the risk-free bond. The reason for this is that asset i_L makes positive payments only when future consumption is low and the marginal utility of consumption is high. The risk-free bond, on the other hand, also makes payments when the the marginal utility of future consumption is low. Because agents prefer the payment structure of asset i_L, they are willing to pay a higher price—accept a lower return—for this asset relative to the risk-free bond. Asset i_L is essentially an insurance policy, and risk-averse agents will pay a premium for insurance.