Economics 702: Macroeconomics II
Spring 2008
Professor Jones

Midterm Examination
March 14, 2008

Instructions. Answer all the questions in your bluebook. There are two questions, and they are weighted equally. You have 120 minutes to complete the exam. Good luck!

1. Consider the following ad hoc macroeconomic model:

$$\begin{aligned} y_t &= p_t - p_t^e, && \text{(AS)} \\ m_t - p_t &= y_t - i_t, && \text{(LM)} \\ y_t &= -\left[i_t - \left(p_{t+1}^e - p_t\right)\right] = p_{t+1}^e - p_t - i_t, && \text{(IS)} \end{aligned}$$

where: y_t, m_t and p_t are the logarithms of output, the money stock and the price level, respectively; i_t is the nominal interest rate; and $p_t^e \equiv E_{t-1}(p_t)$ denotes the mathematical expectation of prices at time t, given the information available at time $t-1$. Note that the equation for the IS curve utilizes the real interest rate (with $p_{t+1}^e \equiv E_t(p_{t+1})$).

   (a) Combine the (IS) and (LM) curves to find a (quasi-) money demand equation, expressing money as a function of the nominal interest rate and expected prices. Briefly interpret the sign of the coefficient on i_t in this equation.

   (b) Now suppose that the monetary authority sets the nominal interest rate using the following variant of the "Taylor Rule":

$$i_t = \alpha y_t + \beta p_t, \quad \alpha, \beta \geq 0. \qquad \text{(TR)}$$

   Combine this rule with the money demand equation to produce a money supply rule. Recalling that full employment output is normalized to equal zero, briefly interpret the coefficients of this money supply rule.

   (c) Combine the preceding equations to eliminate interest rates and output. You should find an equation that expresses the current price level p_t as a function of predicted price levels.

   (d) Find the expected price level, p_t^e. Point out any restrictions you have to impose in deriving your answer.

   (e) Find equilibrium values of prices and output. Briefly interpret.

2. Consider the following application of the Lucas tree model. The preferences of the representative consumer are

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\left[c_t^{1-\sigma}-1\right]\right),\quad 0<\beta<1,\quad \sigma>0.$$

The sole source of the single non-storable good is an everlasting tree that produces d_t units of the consumption good in period t. The economy starts off with each agent owning one tree apiece. The tree's "dividends" follow a two-state Markov process. In particular, d_t takes on the values d_L and $d_H > d_L$. We will also assume that the conditional probabilities are symmetric in that

$$\begin{aligned} f(d_L,d_L) &= \Pr(d_{t+1}=d_L|\, d_t=d_L)=\pi \\ &= f(d_H,d_H), \\ f(d_H,d_L) &= \Pr(d_{t+1}=d_H|\, d_t=d_L)=1-\pi \\ &= f(d_L,d_H). \end{aligned}$$

Let $p_t = p(d_t)$ be the price at time t of a title to all future dividends from a tree. Let $q(d',d)$ be the price of a one-step-ahead contingent claim that delivers one unit of fruit when $d_t = d$ and $d_{t+1} = d'$. Finally, let x_t denote the consumer's financial resources, which she allocates between stocks, contingent claims, and consumption.

(a) Write down the consumer's problem in recursive form and find the first order conditions. Let $z(d')$ denote the "purchasing kernel" that identifies how many state-d' contingent claims the consumer purchases.

(b) Using a recursive approach, define an equilibrium in this economy.

(c) Find the equilibrium contingent claim price $q(d',d_t)$. Express the prices $q_{LL} \equiv q(d_L,d_L)$ and $q_{HL} \equiv q(d_H,d_L)$ as functions of d_L, d_H and π.

(d) Let $R_t^{-1} = R^{-1}(d_t)$ be the time-t price of a risk-free discount bond that pays one unit of consumption at time $t+1$ under any future state. Use the pricing kernel to find $R_L^{-1} \equiv R^{-1}(d_L)$.

(e) Now consider the asset i_L, sold when *current* dividends are low ($d_t = d_L$). If dividends in period $t+1$ are low ($d_{t+1} = d_L$), one unit of asset i_L will deliver $1/\pi$ units of consumption goods. If dividends in period $t+1$ are high, the asset pays nothing.

1. Let p_{iL} denote the price of this asset. Use the pricing kernel to find p_{iL}.
2. What is the expected rate of return on asset i_L?
3. Compare this return to R_L, the return on a risk-free bond when current dividends are low. Which return is higher? Why?