Economics 701: Macroeconomics II
Spring 2008
Professor Jones

**Final Examination**
**Answer Key**

We are considering a version of the stochastic growth model with intermediate goods. Final output is given by

$$Y = \left( \int_0^1 Y(i)^{1/\mu} \, di \right)^{\mu}, \quad \mu > 1. \tag{FPRF}$$

The production function for intermediate goods is given by

$$Y(i) = \widetilde{L}(i)^{\alpha}, \quad \alpha \in (1/2, 1), \tag{PRF}$$

while the total cost of producing good $i$ is

$$\Phi(Y(i)) = WY(i)^{1/\alpha}. \tag{TC}$$

1. (8 points.) A final goods producer operates under perfect competition, solving

$$\max_{[Y(i)]_0^1} \left( \int_0^1 Y(i)^{1/\mu} \, di \right)^{\mu} - \int_0^1 P(i) Y(i) \, di.$$

The first order condition is

$$\mu \left( \int_0^1 Y(i)^{1/\mu} \, di \right)^{\mu - 1} \frac{1}{\mu} Y(i)^{1/\mu - 1} = P(i).$$

Noting that $1/\mu - 1 = (1 - \mu)/\mu$, this reduces to

$$P(i) = \left( \frac{Y}{Y(i)} \right)^{-(1-\mu)/\mu} \Rightarrow Y(i) = P(i)^{\mu/(1-\mu)} Y.$$

2. (8 points.) Intermediate goods producers are price-setters. Inserting the demand function derived in question 1, it follows that producer $i$ solves

$$\max_{P(i)} \Pi(i) = P(i) \times P(i)^{\mu/(1-\mu)} Y - \Phi\left( P(i)^{\mu/(1-\mu)} Y \right)$$

The first order condition is

$$\frac{1}{1-\mu} Y P(i)^{1/(1-\mu)-1} Y = \frac{\partial}{\partial Y(i)} \Phi\left( P(i)^{\mu/(1-\mu)} Y \right) \times \left( \frac{\mu}{1-\mu} \right) Y P(i)^{\mu/(1-\mu)-1}.$$

Assuming $P(i)$ is positive, this reduces to

$$P(i)^{1/(1-\mu)-1-\mu/(1-\mu)+1} = \mu \phi(i),$$

or

$$P(i) = \mu \phi(i),$$

where $\phi(i) \equiv \frac{\partial \Phi(i)}{\partial Y(i)}$ is firm $i$'s marginal cost.

3. (4 points) Fluctuations in this economy are driven by changes in the parameter $\mu$, the log of which follows an AR(1) process:

$$\widehat{\mu}_t \equiv \ln\left(\mu_t/\mu_{ss}\right) = \rho\widehat{\mu}_{t-1} + \varepsilon_t, \quad \mu_{ss} > 0, \quad 0 \leq \rho < 1. \tag{TS}$$

There are a number of reasons why $\mu$ might vary over time. One possibility is changes in the aggregation technology or, equivalently, that final goods producers shift between various aggregation technologies (as described by Wang and Wen). Noting that $\mu$ measures the gross markup (price/cost ratio) imposed by the intermediate goods producer, a third possibility is that changes in $\mu$ reflect changes in the level of competition in each intermediate goods sector. Although we have modelled each intermediate good as having just one producer, it is possible that each intermediate sector is in fact an oligopoly, with a multiple price-setting competitors. In this case, changes in industry structure or behavior could increase or decrease competition, ultimately changing $\mu$.

4. (16 points.) Now consider a symmetric equilibrium, where $Y_t(i) = Y_t(j)$, $\forall i, j, t$.

   (a) In a symmetric equilibrium it follows from equations (FPRF) and (PRF) that

$$\begin{aligned} Y_t &= \left(\int_0^1 Y_t(j)^{1/\mu}\, di\right)^{\mu} = \left(Y_t(j)^{1/\mu}\right)^{\mu} \\ &= Y_t(j) = \widetilde{L}_t(j)^{\alpha} \\ &= \widetilde{L}_t^{\alpha}, \quad \forall j, t. \end{aligned} \tag{SE}$$

   (Note the different indices!)

   (b) Under perfect competition and constant returns, profits in the final goods sector are

$$\Pi_{ft} = \left(\int_0^1 Y_t(i)^{1/\mu}\, di\right)^{\mu} - \int_0^1 P_t(i)\, Y_t(i)\, di = 0.$$

   Using our answer to part (a), this simplifies to

$$Y_t - P_t(j)\, Y_t(j) = Y_t - P_t(j)\, Y_t = 0, \quad \forall j, t,$$

   so that $P_t(j) = 1$, $\forall j, t$. We can then rewrite our answer to question 2 as

$$\phi_t = \mu_t^{-1}.$$

   (c) It follows from equations (TC) and (PRF) that in a symmetic equilibrium:

$$\begin{aligned} \phi_t(i) &= \frac{\partial}{\partial Y_t(i)}\Phi_t(Y_t(i)) = W_t\frac{1}{\alpha}Y_t(i)^{1/\alpha-1} = W_t\frac{1}{\alpha}\left(\widetilde{L}_t(i)^{\alpha}\right)^{1/\alpha} Y_t(i)^{-1} \\ &= W_t\left(\alpha\frac{Y_t}{\widetilde{L}_t}\right)^{-1} = \mu_t^{-1}, \quad \forall i, t. \end{aligned} \tag{MC}$$

5. (14 points.) The consumer's problem can be written as a Lagrangean

$$\begin{aligned} \mathcal{L} = \Bigg\{ E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[ &\ln(C_t) - \chi\left(\frac{1}{1+\gamma}L_t^{1+\gamma} + \frac{1}{1+\gamma}U_t^{1+\gamma}\right) \\ &+ \lambda_t\left((1+r)\,K_t + W_t U_t L_t + \int_0^1 \Pi_t(i)\, di - C_t - K_{t+1}\right)\Bigg]\Bigg\}. \end{aligned}$$

The first-order conditions are

$$\begin{aligned}\frac{1}{C_t} &= \lambda_t,\\ \lambda_t W_t U_t &= \chi L_t^{\gamma},\\ \lambda_t W_t L_t &= \chi U_t^{\gamma},\\ \lambda_t &= \beta E_t\left(\left(1+r\right)\lambda_{t+1}\right).\end{aligned}$$

Combining the two middle equations produces

$$\frac{1}{L_t}U_t^{\gamma} = \frac{1}{U_t}L_t^{\gamma},$$

or

$$U_t = L_t. \tag{UL}$$

Substituting for $\lambda_t$, and imposing $\beta\left(1+r\right) = 1$, we get

$$\frac{1}{C_t} = E_t\left(\frac{1}{C_{t+1}}\right), \tag{EE}$$

$$W_t\frac{1}{C_t} = \chi L_t^{\gamma}\frac{1}{U_t}. \tag{LL}$$

6. (6 points.) The Euler equation for capital was given by equation (EE) above. The capital accumulation equation can be written as

$$K_{t+1} = \left(1+r\right)K_t + Y_t - C_t, \tag{CA}$$

with $Y_t$ following equation (PRF). This constraint can either be derived directly as a resource constraint, or found by combining the household's budget constraint with the definition of profits. In particular, note that in a symmetic equilibrium

$$\Pi_t\left(i\right) = P_t\left(i\right)Y_t\left(i\right) - \Phi_t\left(Y_t\left(i\right)\right) = P_t\left(i\right)Y_t - W_t\widetilde{L}_t\left(i\right) = Y_t - W_t\widetilde{L}_t.$$

Inserting this result into the household's budget constraint yields (CA).

7. (10 points) It follows from equations (SE) and (UL) that

$$Y_t = \widetilde{L}_t^{\alpha} = \left(U_tL_t\right)^{\alpha} = L_t^{2\alpha}. \tag{PRF$'$}$$

Combining equations (MC) and (LL) shows that

$$W_t = \mu_t^{-1}\alpha\frac{Y_t}{\widetilde{L}_t} = C_t\chi L_t^{\gamma}\frac{1}{U_t},$$

Rearranging, and imposing equations (UL) and (PRF$'$), we get

$$\begin{aligned}\frac{\alpha}{\chi}\mu_t^{-1}C_t^{-1} &= \widetilde{L}_tY_t^{-1}L_t^{\gamma}\frac{1}{U_t}\\ &= L_tL_t^{-2\alpha}L_t^{\gamma},\\ \Rightarrow L_t &= \left[\frac{\alpha}{\chi}\mu_t^{-1}C_t^{-1}\right]^{\theta}, \qquad \text{(LL$'$)}\\ \theta &\equiv \frac{1}{1+\gamma-2\alpha}.\end{aligned}$$

8. (12 points.) Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values. It follows from equation (LL$'$) that

$$\exp\left(\widehat{\ell}_t\right) = \frac{L_t}{L_{ss}} = \left(\frac{1-\alpha}{\chi}\right)^{\theta} \mu_t^{-\theta} C_t^{-\theta} / \left[\left(\frac{1-\alpha}{\chi}\right)^{\theta} \mu_{ss}^{-\theta} C_{ss}^{-\theta}\right] = \left[\exp\left(\widehat{\mu}_t + \widehat{c}_t\right)\right]^{-\theta}.$$

Logging both sides yields

$$\widehat{\ell}_t = -\theta\left[\widehat{x}_t + \widehat{c}_t\right]. \tag{LL$''$}$$

With $\gamma \geq 1$ and $0 < \alpha < 1$, $\theta > 0$. Proceeding similarly, it follows from equation (PRF$'$) that

$$\begin{aligned} \exp\left(\widehat{y}_t\right) &= \exp\left(2\alpha\widehat{\ell}_t\right), \\ \widehat{y}_t &= -\lambda\left[\widehat{\mu}_t + \widehat{c}_t\right], \quad \lambda \equiv 2\alpha\theta > 0. \end{aligned} \tag{PRF$''$}$$

The sign on $\widehat{\mu}_t$ is negative because increases in the markup ratio correspond to decreases in output by intermediate goods producers, which reduce aggregate output. To see this, note that with a downward-sloping demand curve, a reduction in output allows price-setting producers to charge a higher price, and with an upward-sloping marginal cost curve, lower output leads to lower marginal cost. Hence, higher markups correspond to lower intermediate goods production.

9. (22 points.) One can express consumption as a function of capital and the markup shock

$$\widehat{c}_t = \eta\widehat{k}_t - \zeta\widehat{\mu}_t, \quad 0 < \zeta < 1. \tag{CF}$$

We will take this as **given**. We compare the log of average labor productivity,

$$\widehat{apl}_t = \widehat{y}_t - \widehat{\ell}_t - \widehat{u}_t,$$

to the log of *measured* labor productivity,

$$\widetilde{apl}_t = \widehat{y}_t - \widehat{\ell}_t.$$

(a) Combining equations (CF) and (PRF$''$) produces

$$\begin{aligned} \widehat{y}_t &= -\lambda\left[\widehat{\mu}_t + \widehat{c}_t\right] = -\lambda\left[\widehat{\mu}_t + \eta\widehat{k}_t - \zeta\widehat{\mu}\right], \\ &= -\lambda\left[\left(1-\zeta\right)\widehat{\mu}_t + \eta\widehat{k}_t\right]. \end{aligned}$$

Note that $1-\zeta$ is positive. Actual labor market productivity is given by

$$\begin{aligned} \widehat{apl}_t &= \widehat{y}_t - \widehat{\ell}_t - \widehat{u}_t \\ &= \widehat{y}_t - 2\widehat{\ell}_t \\ &= -\left(\lambda - 2\theta\right)\left[\widehat{\mu}_t + \widehat{c}_t\right] \\ &= -\left(2\alpha - 2\right)\theta\left[\widehat{\mu}_t + \widehat{c}_t\right] \\ &= 2\left(1-\alpha\right)\theta\left[\left(1-\zeta\right)\widehat{\mu}_t + \eta\widehat{k}_t\right], \end{aligned}$$

with the second line following from equation (UL). Measured labor market productivity is given by

$$
\begin{aligned}
\widetilde{apl}_t &= \widehat{y}_t - \widehat{\ell}_t \\
&= -(\lambda - \theta)[\widehat{\mu}_t + \widehat{c}_t] \\
&= -(2\alpha - 1)\theta[\widehat{\mu}_t + \widehat{c}_t] \\
&= -(2\alpha - 1)\theta\left[(1-\zeta)\widehat{\mu}_t + \eta\widehat{k}_t\right].
\end{aligned}
$$

(b) With $\alpha \in (1/2, 1)$, the answer to part (a) shows that when $\widehat{\mu}$ rises, output falls, average labor productivity rises, and measured labor productivity falls. Extrapolating from the initial response, one would expect average labor productivity to move in the direction opposite of output, so that it is counter-cyclical, while measured labor productivity is pro-cyclical.

(c) Intuitively, an increase in the markup causes $Y$ and $LU$ to both decrease. But with decreasing returns to effective labor, a decrease in labor causes average productivity, $Y/(LU)$, to increase. On the other hand, measured productivity accounts for the part of output decrease attributable to lower hours, but not for the part attributable to lower effort. Since the proportional decrease in output due to the combination of lower hours and effort exceeds the decrease in hours alone, measured productivity decreases.