Economics 701: Macroeconomics II
Spring 2008
Professor Jones

Final Examination
May 8, 2008

Instructions. Answer all the questions in your bluebook. You have 120 minutes to complete the exam. Good luck!

(Inspired by Wang and Wen, 2006b.) Consider the following simplified version of a stochastic growth model. The population and steady state technology level are constant and normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

Letting $i \in [0,1]$ index intermediate goods, final output is given by

$$Y = \left(\int_0^1 Y(i)^{1/\mu} \, di \right)^{\mu}, \quad \mu > 1. \tag{FPRF}$$

(When possible, time subscripts are omitted.) Final goods are produced under perfect competition, with each producer solving

$$\max_{[Y(i)]_0^1} Y - \int_0^1 P(i) Y(i) \, di,$$

subject to equation (FPRF).

Intermediate goods producers, on the other hand, are price setters. The production function for intermediate goods is given by

$$Y(i) = \widetilde{L}(i)^{\alpha}, \quad \alpha \in (1/2, 1), \tag{PRF}$$

where $\widetilde{L}$ is effort-weighted labor. Letting W denote the real wage, equation (PRF) then implies that the total cost of producing good i is

$$\Phi(Y(i)) = W\widetilde{L}(i) = WY(i)^{1/\alpha}. \tag{TC}$$

Producer i thus solves

$$\max_{P(i)} \Pi(i) = P(i) \times Y_i[P(i)] - \Phi(Y_i[P(i)]),$$

where $\Pi(i)$ is producer i's profits, and $Y_i[P(i)]$ is the demand function for good i.

1. (8 points.) Solving the maximization problem for a final goods producer, show that the intermediate goods quantity $Y(i)$ can be written as a function of the intermediate price $P(i)$ and the aggregate quantity Y.

2. (8 points) Using your answer to question 1, solve the maximization problem for an intermediate goods producer. In particular, show that

$$P(i) = \mu\phi(i),$$

where $\phi(i) \equiv \frac{\partial \Phi(i)}{\partial Y(i)}$ is firm i's marginal cost.

3. (4 points) Fluctuations in this economy are driven by changes in the parameter μ, the log of which follows an AR(1) process:

$$\widehat{\mu}_t \equiv \ln(\mu_t/\mu_{ss}) = \rho\widehat{\mu}_{t-1} + \varepsilon_t, \quad \mu_{ss} > 1, \quad 0 \leq \rho < 1. \tag{TS}$$

Note that μ measures the gross markup (price/cost ratio) imposed by the intermediate goods producer. Why might μ vary over time?

4. (16 points.) Now consider a symmetric equilibrium, where $Y_t(i) = Y_t(j), \forall i, j, t$.

   (a) Show that in a symmetric equilibrium

   $$Y_t(i) = Y_t = \widetilde{L}_t^{\alpha}. \tag{SE}$$

   (b) Note that final goods are the numeraire, so that $P_t = 1$. Using the profit function of final goods producers, show that

   $$P_t(i) = P_t = 1.$$

   (c) Show that in a symmetric equilibrium:

   $$\phi_t = W_t\left(\alpha\frac{Y_t}{\widetilde{L}_t}\right)^{-1} = \mu_t^{-1}. \tag{MC}$$

Now let's consider consumers. The preferences of the representative household over consumption, C_t, labor hours, L_t, and effort, U_t, are given by

$$E_0\left(\sum\nolimits_{t=0}^{\infty}\beta^t\left[\ln(C_t) - \chi\left(\frac{1}{1+\gamma}L_t^{1+\gamma} + \frac{1}{1+\gamma}U_t^{1+\gamma}\right)\right]\right),$$

$$0 < \beta < 1, \quad \gamma \geq 1, \quad \chi > 0.$$

Households receive labor income ($W_t\widetilde{L}_t = W_tU_tL_t$) and profits from firms. Households earn a gross return of $(1+r)K_t$ on their assets, K_t, with $\beta(1+r) = 1$. Assume that assets held at the beginning of period $t+1$, K_{t+1}, are chosen in period t. Note that in this economy, capital is used only as a storage device, and not as a factor of production. Households spend their income on consumption and investment in capital. Households also face the usual initial, non-negativity and No-Ponzi-Game conditions.

5. (14 points.) Find the first order conditions for utility maximization. Show that effort and hours are connected by

$$U_t = L_t. \tag{UL}$$

6. (6 points.) Imposing equilibrium, find: the Euler equation for capital; and the capital accumulation equation/resource constraint.

7. (10 points) Imposing (symmetric) equilibrium, show that

$$Y_t = L_t^{2\alpha}. \tag{PRF$'$}$$

Eliminate wages and effort to express labor hours (L_t, **not** $\widetilde{L}_t$) as a function of consumption and the markup ratio μ_t.

8. (12 points) Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values. Using your answer to question 7, show that output and hours can be approximated as

$$\widehat{\ell}_t = -\theta\left[\widehat{\mu}_t + \widehat{c}_t\right], \quad \theta > 0, \tag{LL$''$}$$

$$\widehat{y}_t = -\lambda\left[\widehat{\mu}_t + \widehat{c}_t\right], \quad \lambda > 0. \tag{PRF$''$}$$

with ℓ denoting the logarithm of L, not $\widetilde{L}$. Why are the coefficients on the markup ratio $\widehat{\mu}_t$ negative?

9. (22 points) After log-linearizing the Euler and Capital Accumulation equations, and solving the resulting system (which includes equation (TS)) by the method of undetermined coefficients, one can express consumption as a function of capital and the markup shock

$$\widehat{c}_t = \eta\widehat{k}_t - \zeta\widehat{\mu}_t, \quad 0 < \zeta < 1. \tag{CF}$$

(Take this as **given**.)

Consider the log of average labor productivity,

$$\widehat{apl}_t = \widehat{y}_t - \widehat{\ell}_t - \widehat{u}_t,$$

and the log of *measured* labor productivity,

$$\widetilde{apl}_t = \widehat{y}_t - \widehat{\ell}_t$$

(a) Express $\widehat{y}_t$, $\widehat{apl}_t$ and $\widetilde{apl}_t$ as functions of capital and the markup ratio. (**Hint:** Recall equation (UL).)

(b) Suppose that the economy experiences an unexpected increase in the markup ratio ($\widehat{\mu}$ *rises*). In what direction will output, average labor productivity and measured labor productivity initially respond? Extrapolating from the initial response, would you expect labor productivity to be pro- or counter-cyclical in this model? How about measured labor productivity?

(c) Explain intuitively your answer to part (b).