Economics 701: Macroeconomics II
Spring 2007
Professor Jones

Midterm Examination
March 13, 2007

Instructions. Answer all the questions in your bluebook. There are two questions, and they are weighted equally. You have 90 minutes to complete the exam. Good luck!

1. (From an exam by Prof. Lars Ljungqvist) Let y_t, w_t, m_t and p_t be the logarithms of output, the wage level, the money stock and the price level, respectively. Consider the following ad hoc macroeconomic model:

$$y_t = m_t - p_t - \theta_t, \tag{AD}$$
$$y_t = p_t - w_t + \eta_t, \tag{AS}$$
$$w_t = E\left(p_t | I_{t-1}^p\right), \tag{WS}$$
$$m_t = \alpha\eta_t. \tag{MS}$$

where: θ_t is an exogenous demand shock (the minus sign in front of θ_t is intentional); η_t is an exogneous supply shock; and $E\left(p_t | I_{t-1}^p\right)$ denotes mathematical expectations conditioned on the private sector's information set, I_{t-1}^p. This information set consists of all variables dated $t-1$ and earlier:

$$I_{t-1}^p = \left\{y_{t-j}, p_{t-j}, m_{t-j}, w_{t-j}, \theta_{t-j}, \eta_{t-j}\right\}_{j=1}^{\infty}.$$

Equation (MS) implicitly assumes, however, that the monetary authority also observes the supply shock η_t:

$$I_{t-1}^g = \left\{I_{t-1}^p, \eta_t\right\}.$$

The random shocks θ_t and η_t are both zero-mean and i.i.d., with

$$V(\theta_t) = \sigma_\theta^2; \quad V(\eta_t) = \sigma_\eta^2.$$

In addition, θ_t and η_t are independent of each other at all leads and lags.

   (a) Find reduced-form expressions for output and prices as functions of m_t, θ_t, η_t and $E\left(p_t | I_{t-1}^P\right)$. Then solve for $E\left(p_t | I_{t-1}^P\right)$.

   (b) Assume that the monetary authority sets α to minimize the variance of output. Let y_t^U denote the expression for output that arises under this monetary rule. Find the optimizing value of α and y_t^U.

   (c) Now suppose that the monetary authority still observes η_t while the private sector does not, but that the monetary authority announces its money supply decision before wages are set, so that

$$I_{t-1}^p = \left\{\left\{y_{t-j}, p_{t-j}, m_{t-j}, w_{t-j}, \theta_{t-j}, \eta_{t-j}\right\}_{j=1}^{\infty}, m_t\right\}.$$

   (Equation (MS) still holds.) Let y_t^A denote the expression for output that arises under this monetary regime. Solving again for $E\left(p_t | I_{t-1}^P\right)$, find y_t^A.

(d) Now suppose that the monetary authority observes the demand shock θ_t after it sets m_t but prior to making any announcements. Suppose further that the monetary authority selectively announces m_t as way to further reduce the variance of output. Assume for the moment that

$$y_t = \begin{cases} y_t^A, & \text{when } m_t \text{ is announced} \\ y_t^U, & \text{when } m_t \text{ is not announced} \end{cases}. \tag{ANN}$$

For what values of θ_t and η_t should the monetary authority announce m_t?

(e) Is the combination of equation (ANN) and your answer to part (d) consistent with equation (WS)?

2. (Extended from a question by Prof. Lars Ljungqvist) Consider the following variant of the Lucas tree model. The preferences of the representative agent are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(c_t^{1-\sigma} - 1 \right) \right), \quad 0 < \beta < 1, \quad \sigma > 0,$$

where c_t denotes consumption. Each tree provides a constant flow of d units of non-storable "fruit", or dividends, per period. The economy starts off with each household owning one such tree. Let p_t be the price at time t of a title to all future dividends from a tree.

Each period, the government collects consumption taxes on the agent, so that the agent's total expenditures on consumption are $(1+\tau_t)\, c_t$. The tax rate τ_t is an exogenous random variable, following a Markov process with the stationary one-step transition density $f(\tau', \tau)$. All revenues from the consumption tax are refunded to the agent as a lump sum transfer, T_t, which the agent treats a form of revenue. It follows that in equilibrium

$$T_t = \tau_t c_t.$$

However, the representative agent takes both T_t and τ_t as given.

Let $R_t^{-1} = R^{-1}(\tau_t)$ be the time-t price of a risk-free discount bond that pays one unit of consumption at time $t+1$. Finally, let x_t denote the consumer's *post-transfer* financial resources, which she allocates between risk-free bonds, stocks, consumption and consumption taxes.

(a) Write down the consumer's problem in recursive form and find the first order conditions.

(b) Using a recursive approach, define an equilibrium in this economy. (**Hint:** make sure you account for the government.)

(c) Derive the equilibrium pricing function for bonds, $R_t^{-1} = R^{-1}(\tau_t)$ and the risk-free interest rate $R_t = R(\tau_t)$.

(d) It is often argued that imposing a tax on consumption will increase the willingness of consumers to save, leading to lower interest rates. Suppose that the consumer faces a constant tax rate τ. How does increasing τ affect the risk-free rate R_t? Briefly explain.

(e) Define the return on stocks, R_t^S, by

$$R_t^S = \frac{p_{t+1} + d_{t+1}}{p_t},$$

and define the equity premium, e_t, by $e_t = E_t\left(R_t^S\right) - R_t$. Find e_t, and briefly explain what it means.