Economics 701: Macroeconomics II
Spring 2007
Professor Jones

Final Examination
May 16, 2007

Instructions. Answer all the questions in your bluebook. You have 120 minutes to complete the exam. Good luck!

Consider the following simplified version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_{Mt}\geq 0} \ \Pi_t = Y_t - W_t L_{Mt},$$
$$Y_t = \frac{1}{1-\alpha} L_{Mt}^{1-\alpha}, \quad 0<\alpha<1, \tag{PRF}$$

where: Π_t is profit; Y_t is market output; L_{Mt} is market labor; and W_t is the real wage. There is no population growth, and the population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_{Mt}\right) + Z_t \ln\left(C_{Ht}\right) - \frac{\chi}{1+\gamma}\left(L_{Mt}+L_{Ht}\right)^{1+\gamma} \right] \right),$$
$$0<\beta<1, \quad \gamma \geq 0, \quad \chi > 0,$$

where: C_{Mt} denotes consumption of market goods; C_{Ht} denotes consumption of home-produced goods; Z_t is a taste shifter; K_t denotes assets; L_{Mt} denotes time spent in firm (market) production; and L_{Ht} denotes time spent in home production.

Households face two budget constraints. The first is the budget constraint for market consumption and assets, which takes the standard form:

$$C_{Mt} + K_{t+1} = (1+r) K_t + W_t L_{Mt} + \Pi_t, \tag{FBC}$$

with $\beta(1+r) = 1$. Note that in this economy, capital is used only as a storage device, and not as a factor of production. The second budget constraint is the constraint for home-produced goods:

$$C_{Ht} = \frac{1}{1-\eta} L_{Ht}^{1-\eta}, \quad 0<\eta<1. \tag{HP}$$

Households also face the usual initial, non-negativity and No-Ponzi-Game conditions.

Finally, the log of the taste shifter Z_t follows an AR(1) process

$$\widehat{z}_t \equiv \ln\left(Z_t/Z_{SS}\right) = \phi \widehat{z}_{t-1} + \varepsilon_t, \quad Z_{ss} > 0, \quad 0 \leq \phi < 1, \tag{TS}$$

where $\{\varepsilon_t\}$ is an exogenous stationary martingale difference sequence.

1. (12 points) Assuming interiority, find the first order conditions for utility maximization. You should find two time allocation conditions and an Euler equation.

2. (8 points) Find the first order condition for profit maximization. Imposing labor market equilibrium, show that

$$L_{Ht} = Z_t (1 - \eta) C_{Mt} L_{Mt}^{\alpha}. \tag{HM}$$

3. (8 points) Using equation (HM), find: the market labor allocation condition; the Euler equation; and the capital accumulation equation. Your answer should contain no home production variables.

4. (38 points) Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values.

   (a) Show that log-linearized expressions for market labor and output are

$$\begin{aligned} \widehat{\ell}_{Mt} &= -\left[\lambda_1 \widehat{z}_t + \theta_1 \widehat{c}_{Mt}\right], \quad \lambda_1, \theta_1 > 0, \\ \widehat{y}_t &= -\left[\lambda_2 \widehat{z}_t + \theta_2 \widehat{c}_{Mt}\right], \quad \lambda_2, \theta_2 > 0. \end{aligned}$$

   To simplify your algebra, let $L_t = L_{Mt} + L_{Ht}$ denote total labor hours, and let L_{SS} denote its steady-state value.

   (b) Why are λ_1 and θ_1 both positive? Explain intuitively.

   (c) Consider the steady-state ratio $f_{SS} \equiv L_{HSS}/L_{SS}$, the fraction of labor hours devoted to home production. Is λ_1 increasing or decreasing in this ratio? Provide an intuitive explanation.

   (d) Given your answer to part (c), would you expect home production shocks to have bigger effects in fully-industrialized countries, or in developing countries? Briefly explain.

5. (18 points) Suppose that the steady state consumption-to-capital ratio, C_{MSS}/K, is ψ, with $\psi > r$. Using this fact, it is straightforward, if tedious, to show that the log-linearized capital accumulation equation is

$$\begin{aligned} \widehat{k}_{t+1} &= (1+r)\,\widehat{k}_t - \omega_1 \widehat{z}_t - \omega_2 \widehat{c}_{Mt}, \\ \omega_2 &> \omega_1 > 0. \end{aligned} \tag{CA$'$}$$

   It is also straightforward to show that the log-linearized Euler equation is:

$$E_t\left(\widehat{c}_{M,t+1}\right) = \widehat{c}_{Mt}. \tag{EE$'$}$$

   (You can take both (CA′) and (EE′) as given: do not derive them.) It can be shown that the system given by equations (CA′), (EE′) and (TS) is saddle-path stable. Solve this system with the method of undetermined coefficients. In particular, assume that consumption can be written as a function of capital and technology:

$$\begin{aligned} \widehat{c}_{Mt} &= \pi \widehat{k}_t - \mu \widehat{z}_t, \\ \pi, \mu &> 0, \end{aligned} \tag{CF}$$

   and proceed as follows:

(a) Insert equation (CF) into equation (CA′) to express $\widehat{k}_{t+1}$ as a function of $\widehat{k}_t$ and $\widehat{z}_t$. Call this equation (CA″).

(b) Insert equation (CF) into the *right*-hand-side of equation (EE′) to express $E_t\left(\widehat{c}_{M,t+1}\right)$ as a function of $\widehat{k}_t$ and $\widehat{z}_t$.

(c) Insert equation (CF) into the *left*-hand-side of equation (EE′) to express $E_t\left(\widehat{c}_{M,t+1}\right)$ as a function of $E_t\left(\widehat{k}_{t+1}\right)$ and $E_t\left(\widehat{z}_{t+1}\right)$. Then insert (CA″) and (TS) to express $E_t\left(\widehat{c}_{M,t+1}\right)$ as a function of $\widehat{k}_t$ and $\widehat{z}_t$.

(d) Compare the two expressions for $E_t\left(\widehat{c}_{M,t+1}\right)$. If the two expressions are to be identical, what must be the values of π and μ?

6. (16 points) Consider the log of average market labor productivity, $\widehat{apl}_t = \widehat{y}_t - \widehat{\ell}_{Mt}$.

(a) Express $\widehat{\ell}_{Mt}$ and $\widehat{apl}_t$ as functions of capital and preferences for home-produced goods.

(b) Suppose that the economy experiences an unexpected increase in the demand for home-produced goods ($\widehat{z}$ *rises*). In what direction will market output, market labor, and average productivity initially respond? Extrapolating from the initial response, would you expect average market labor productivity to be pro- or counter-cyclical in this model? Explain briefly. Graphs are nice but unnecessary.