Economics 701: Macroeconomics II
Spring 2006
Professor Jones

Midterm Examination
March 25, 2006

Instructions. Answer all the questions in your bluebook. There are two questions, and they are weighted equally. You have 105 minutes to complete the exam. Good luck!

1. (Based on Taylor, 1979, 1980.) Let y_t, w_t, m_t and p_t be the logarithms of output, the wage level, the money stock and the price level, respectively. Consider the following ad hoc macroeconomic model:

$$y_t = m_t + \theta_t - p_t, \tag{AD}$$
$$y_t = p_t - \frac{1}{2}w_t - \frac{1}{2}w_{t-1}, \tag{AS}$$
$$w_t = E\left(p_t | I_{t-1}\right), \tag{WS}$$

where $E\left(p_t | I_{t-1}\right)$ denotes mathematical expectations conditioned on the information set I_{t-1}. The presence of two wages reflects the notion that changing wages is costly, so that agents renegotiate wages every two years; at any time t, only half the agents have updated their wages.

The demand shock θ_t follows an MA(1) process:

$$\theta_t = \varepsilon_t + \phi\varepsilon_{t-1}, \tag{TS}$$

where ε_t is an exogenous i.i.d. variable, with $E\left(\varepsilon_t\right) = 0$ and $V\left(\varepsilon_t\right) = \sigma_\varepsilon^2$.

The information set I_t consists of all variables dated t and earlier:

$$I_t = \left\{y_{t-j}, p_{t-j}, m_{t-j}, \theta_{t-j}, \varepsilon_{t-j}, w_{t-j}\right\}_{j=0}^{\infty}.$$

Finally the monetary authority sets the money supply according to

$$m_t = \beta\varepsilon_{t-1}. \tag{MS}$$

(a) Find reduced-form expressions for output and prices as functions of m_t, θ_t, $E\left(p_t | I_{t-1}\right)$ and $E\left(p_{t-1} | I_{t-2}\right)$.

(b) Find $w_t = E\left(p_t | I_{t-1}\right)$.

1. First, solve for $E\left(p_t | I_{t-1}\right)$. (**Hint:** Be careful in how you apply the law of iterated expectations.)
2. Second, use $w_t = E\left(p_t | I_{t-1}\right)$ to show that w_t follows a first-order difference equation.
3. Third, solve this difference equation to express w_t in terms of ε_t and/or its lagged values.

(c) Use your answer to part (b) to express output and prices as functions of ε_t, ε_{t-1} and (if possible) w_t.

(d) Note that the shock ε_t affects aggregate demand (shown in equation (AD)) for only two periods, while your answers to parts (b) and (c) should show that the shock can affect output and prices for many periods. Does this make intuitive sense to you? Why?

(e) What value of β minimizes the variance of output?

2. Consider the following application of the Lucas tree model. The preferences of the representative consumer are

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\left[c_t^{1-\sigma}-1\right]\right),\quad 0<\beta<1,\quad \sigma>0.$$

The sole source of the single non-storable good is an everlasting tree that produces d_t units of the consumption good in period t. The economy starts off with each agent owning one tree apiece. The tree's "dividends" follow a two-state Markov process. In particular, d_t takes on the values d_L and $d_H > d_L$. We will also assume that the conditional probabilities are symmetric in that

$$\begin{aligned}
f(d_L,d_L) &= \Pr\left(d_{t+1}=d_L|\, d_t=d_L\right)=\pi \\
&= f(d_H,d_H), \\
f(d_H,d_L) &= \Pr\left(d_{t+1}=d_H|\, d_t=d_L\right)=1-\pi \\
&= f(d_L,d_H).
\end{aligned}$$

Let $p_t = p(d_t)$ be the price at time t of a title to all future dividends from a tree. Let $q(d',d)$ be the price of a one-step-ahead contingent claim that delivers one unit of fruit when $d_t = d$ and $d_{t+1} = d'$. Finally, let x_t denote the consumer's financial resources, which she allocates between stocks, contingent claims, and consumption.

(a) Write down the consumer's problem in recursive form and find the first order conditions. Let $z(d')$ denote the "purchasing kernel" that identifies how many state-d' contingent claims the consumer purchases.

(b) Find the equilibrium contingent claim price $q(d',d_t)$. Consider the prices $q(d_L,d_H)$ and $q(d_H,d_L)$. Which contingent claim has the higher price. Why?

(c) Derive the equilibrium pricing function for stocks, $p(d_t)$. (This function should not include any expected future prices.)

(d) Let $R_t^{-1} = R^{-1}(d_t)$ be the time-t price of a risk-free discount bond that pays one unit of consumption at time $t+1$ under any state. Use the pricing kernel to find $R^{-1}(d_t)$. Express the bond price that is in effect when dividends are low, $R^{-1}(d_L)$, as a function of d_L, d_H and π. Is this price greater than or less than β? Why?